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Tornado-Borne Missile Speeds

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U.S. DEPARTMENT OF COMMERCE, Elliot L. Richardson, *Secretary*
James A. Baker, III, *Under Secretary*
Dr. Betsy Ancker-Johnson, *Assistant Secretary for Science and Technology*
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Acting Director*

Tornado - Borne Missile Speeds

Emil Simiu and Martin Cordes

At the request of the United States Nuclear Regulatory Commission (NRC) the National Bureau of Standards (NBS) has carried out an independent investigation into the question of tornado-borne missile speeds, with a view to assisting NRC in identifying pertinent areas of uncertainty and in estimating credible tornado-borne missile speeds - within the limitations inherent in the present state of the art. The investigation consists of two parts: (1) a study, covered in this report, in which a rational model for the missile motion is proposed, and numerical experiments are carried out corresponding to various assumptions on the initial conditions of the missile motion, the structure of the tornado flow, and the aerodynamic properties of the missile; (2) a theoretical and experimental study of tornado-borne missile aerodynamics, conducted by Colorado State University (CSU) under contract with NBS, to be covered in a separate report by CSU. In the present report, the factors affecting missile motion, and their influence upon such motion, are examined. Information is provided on a computer program developed for calculating missile speeds. Maximum speeds for a number of specified potential tornado-borne missiles are presented, corresponding to a set of assumptions believed by the writers to be reasonable for design purposes. It is pointed out that higher speeds are conceivable if it is assumed that certain circumstances, examined in the body of the report, will obtain. It is the judgment of the writers that the probabilities of occurrence of such higher speeds for any given tornado strike are low. More than qualitative estimates of such probabilities are, however, beyond the scope of this investigation.

KEY WORDS: Missiles; nuclear engineering; structural engineering; tornadoes; wind.

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TABLE OF CONTENTS

LIST OF SYMBOLS

A	= effective area
A_1, A_2, A_3	= projected areas
c	= coefficient in Eq. 25
C	= coefficient in Eq. 12
C_D	= effective drag coefficient
$C_{D_1}, C_{D_2}, C_{D_3}$	= drag coefficients corresponding to areas A_1, A_2, A_3, respectively
C_L	= lift coefficient
d	= characteristic dimension of body
$\overline{D}$	= drag force
g	= acceleration of gravity
h	= vertical displacement
$\overline{i}, \overline{j}, \overline{k}$	= unit vectors
k_1, k_2, k_3	= parameters of the tornado flow
m	= mass of missile
r	= radius (distance from vortex center)
R_m	= value of r at which the horizontal velocity in the vortex flow maximum
$R^*(z)$	= radius beyond which radial and vertical velocity components vanish
Re	= Reynolds number
t	= time
U	= velocity
V	= fluid velocity relative to the body
v_H max	= maximum horizontal missile speed
v_m	= maximum horizontal wind velocity in vortex flow
v_M	= missile velocity, with components $v_{M_x}, v_{M_y}, v_{M_z}$
v_{rot}	= horizontal wind velocity in vortex flow
v_R	= radial wind velocity in vortex flow
v_{torn}	= $v_m + v_T$
v_T	= translation velocity of tornado
$\overline{v}_w$	= wind velocity

v_z = vertical wind velocity

v_θ = tangential wind velocity in vortex flow

v_{θ_m} = maximum tangential velocity

w = weight of missile

x = coordinate axis

y = coordinate axis

z = coordinate axis

α = angular displacement

ν = kinematic viscosity

ρ = air density

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

To ensure the safety of nuclear power plants in the event of a tornado strike it is required that, in addition to the direct action of the wind and of the moving ambient pressure field, the designer consider the impact of tornado-borne missiles, i.e., of objects moving under the action of aerodynamic forces induced by the tornado wind. It is, therefore, necessary that estimates be made of the speeds attained by potential missiles under tornado wind conditions specified for the design of nuclear power plants.

Several such estimates have been reported so far [1, 2, 3, 4, 5, 6, 7]. In certain instances the differences between various estimates are very large; for example, the predicted speeds of a utility pole [13-in (0.33 m) diameter and a 35-ft (10.7 m) length] predicted by Refs. 5, 6, 1, 2, 4 and 3 are 16.5 m/s, 30.5 m/s, 41.2 m/s, 42.7 m/s, 52.2 m/s and 54.9 m/s, respectively. For any given missile, the kinetic energy associated with translational motion is proportional to the square of the speed; in the case of the utility pole, therefore, the ratio of the largest to the smallest of the kinetic energy estimates of Ref. 1 through Ref. 6 is greater than 10. For other missiles, the ratio varies from almost 2 in the case of a 1-in (2.54 cm) diameter steel rod to over 5 in the case of a 4000 lb (18,000N) standard automobile.

In most cases, such large discrepancies are the consequence of differences between basic assumptions used in the various estimation procedures. These assumptions include:

- the initial conditions of the problem, i.e., the initial position of the object with respect to the ground and to the tornado center, and the initial velocity of the object.

- the detailed features of the wind flow field.

- the aerodynamic characteristics of the object (which, in most cases of practical interest, is a bluff body).

Differences between the various sets of basic assumptions used in estimating tornado-borne missile velocities may be ascribed, in part, to the probabilistic nature of the problem. Indeed, for any given tornado wind field, the initial conditions constitute a set or random variables with a very large number of possible values, the choice of which is not unique. More importantly, however, such differences are a consequence of serious uncertainties regarding both the structure of the tornado flow and the aerodynamic behavior of the potential missile.

At the request of the United States Nuclear Regulatory Commission (NRC), the National Bureau of Standards has carried out an independent investigation into the question of tornado-borne missile speeds, with a view to assisting NRC in identifying pertinent areas of uncertainty and in estimating credible tornado-borne missile speeds--within the limitations inherent in the current state of the art.

The objectives of the investigation were defined as follows:

1. To select a rational model for the tornado-borne missile motion.
2. To develop a computer program based on this model, capable of computing missile trajectories and velocities for any specified initial conditions, tornado wind speed model, and assumptions regarding the drag force acting on the missile.
3. To calculate, for a specified set of initial conditions and for a specified wind speed model, the trajectories and velocities corresponding to a number of specified potential missiles.
4. To determine, in a number of representative cases, the sensitivity of the calculated results to changes in the assumed initial conditions or in the assumed tornado wind speed model.
5. To obtain and interpret information, based on wind tunnel tests, regarding the aerodynamic behavior of various potential missiles, i.e., drag coefficients for various missile motions, including motion in the tumbling mode.
6. From the results of items 3, 4 and 5, suggest credible speeds of selected potential tornado-borne missiles, compatible with the current state of the art in nuclear power plant design.

2. MODEL FOR THE TORNADO-BORNE MISSILE MOTION

The motion of an object may be described in general by solving a system consisting of three equations of balance of momenta and three equations of balance of moments of momenta. In the case of a bluff body, one major difficulty in writing these six equations is that the aerodynamic forcing functions are not known.

It is possible to measure in the wind tunnel aerodynamic forces and moments acting on a bluff body under static conditions for a sufficient number of positions of the body with respect to the mean direction of the flow. On the basis of such measurements, the dependence of the forces and moments on position, and corresponding aerodynamic derivatives, can be obtained. Aerodynamic forces and moments can then be calculated following the well-known pattern used in airfoil theory; for example, if an airfoil has a time-dependent vertical motion h(t) in a uniform flow with velocity U, and if the angle of attack is α = const, the lift coefficient is

$$C_L = \frac{dC_L}{d\alpha}\left(\alpha + \frac{1}{U}\frac{dh}{dt}\right) \qquad (1)$$

This procedure for calculating aerodynamic forces and moments is valid if the quasi-steady assumption [Ref. 8, p. 192] is acceptable and if the body concerned behaves

aerodynamically like an airfoil - i.e., if the body is streamlined and if no flow separation occurs. However, in the case of unconstrained bluff bodies moving in a wind flow the validity of such a procedure remains to be demonstrated.

In the absence of a satisfactory model for the aerodynamic description of the missile as a rigid (six-degrees-of-freedom) body, it is customary to resort to the alternative of describing the missile as a material point acted upon by a drag force

$$\overline{D} = 1/2\ \rho\ C_D A \left| \overline{v}_w - \overline{v}_M \right| (\overline{v}_w - \overline{v}_M) \tag{2}$$

where ρ = air density, $\overline{v}_w$ = wind velocity, $\overline{v}_M$ = missile velocity, A is a suitably chosen area and C_D is the corresponding drag coefficient.

This model is reasonable if, during its motion, the missile either (a) maintains a constant or almost constant attitude with respect to the relative velocity vector $\overline{v}_w - \overline{v}_M$, or (b) has a tumbling motion such that, with no significant errors, some mean value of the quantity C_DA can be used in the expression for the drag $\overline{D}$. The assumption of a constant body attitude with respect to the flow would be credible if the aerodynamic force were applied at all times exactly at the center of mass of the body--which is highly unlikely in the case of a bluff body in a tornado flow--, or if the body rotation induced by a non-zero aerodynamic moment with respect to the center of mass were inhibited by aerodynamic damping forces intrinsic in the body-fluid system. The question thus arises as to whether such stabilizing forces may be expected to be present.

It is of interest at this point to mention certain experimental results--obtained in studies of bridge deck aerodynamic stability--which provide useful insights into the question at hand. Consider a body restrained by four springs of equal stiffness, immersed in a horizontal flow (Fig. 1), and subjected to an impulse which produces angular oscillations $\theta(t)$ about the position of equilibrium [9]. In the case of an airfoil with a sufficiently small angle of attack so that flow separation does not occur, the aerodynamic damping, which is proportional to the quantity denoted by H_2^* in Ref. 9, is positive. This implies that the flow will contribute , along with the viscous damping inherent in the springs, to the damping out of the oscillations. On the other hand, for bluff bodies, at high velocities of the flow and for vanishing values of the spring stiffness, the aerodynamic damping is negative in the large majority of the cases tested [9].

These results suggest that, in general, no stabilizing effect by the flow can be expected to inhibit the tumbling of bluff bodies. The assumption that potential tornado-borne missiles will tumble during their motion appears therefore to be reasonable. It will be this assumption that will be used in this work.

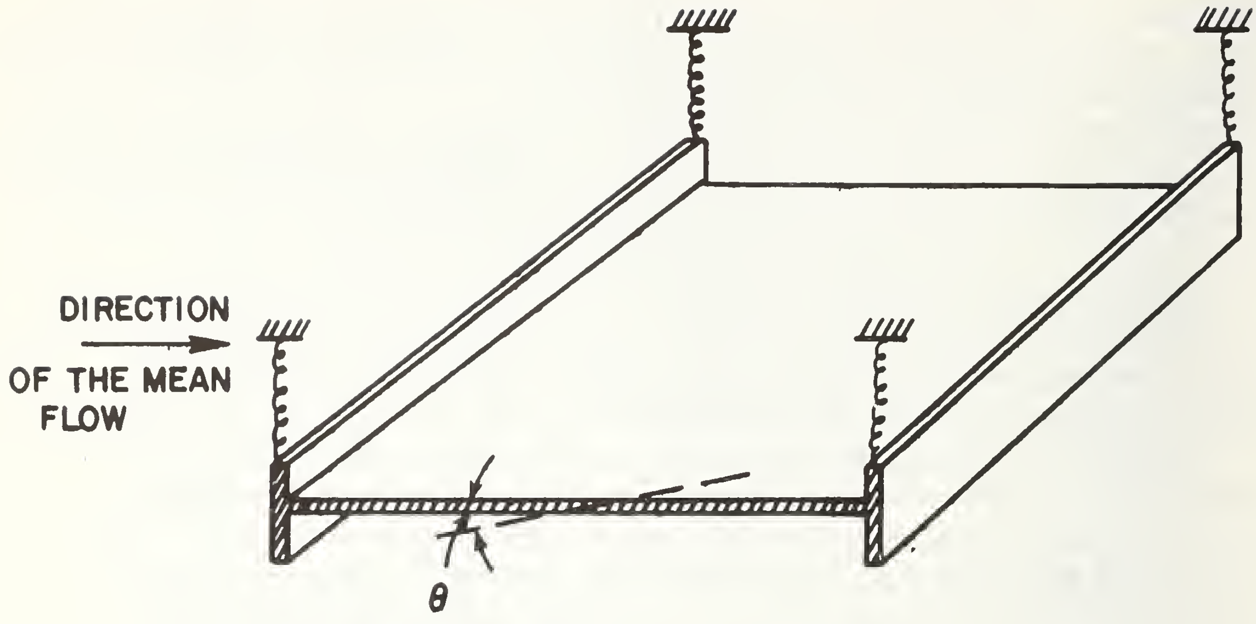

Figure 1 Bluff Bridge Deck Section

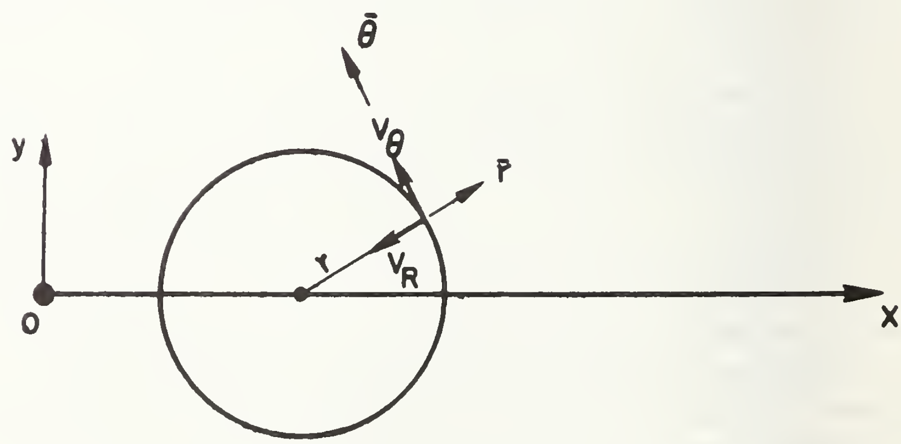

Figure 2 Notations

Assuming that Eq. 2 is valid and that the average lift force vanishes under tumbling conditions, the motion of the missile viewed as a one-degree-of-freedom system is governed by the relation:

$$\frac{d\bar{v}_M}{dt} = -1/2\,\rho\,\frac{C_D A}{m}\,\left|\bar{v}_M - \bar{v}_w\right|\,(\bar{v}_M - \bar{v}_w) - g\bar{k} \tag{3}$$

where g = acceleration of gravity, $\bar{k}$ = unit vector along the vertical axis and m = mass of missile.

It follows from Eq. 3 that for a given flow field and initial conditions, the motion depends only upon the value of the parameter $C_D A/m$. Throughout this work, the area, A, will be expressed in m^2 and the mass, m, in kg. To transform the parameter $C_D A/w$, where A is expressed in ft^2 and the weight in lb, into the parameter $C_D A/m$, where A is expressed in m^2 and the mass, m, in kg, the following relations are used:

$$\frac{C_D A}{w} = 1\,\frac{ft^2}{lb} \quad \rightarrow \quad \frac{C_D A}{m} = .205\,\frac{m^2}{kg} \tag{4a}$$

$$\frac{C_D A}{m} = 1\,\frac{m^2}{kg} \quad \rightarrow \quad \frac{C_D A}{w} = 4.902\,\frac{ft^2}{lb} \tag{4b}$$

3. COMPUTER PROGRAM FOR CALCULATING TORNADO-BORNE MISSILE TRAJECTORIES AND VELOCITIES

To calculate and plot trajectories and velocities of tornado-borne missiles, a computer program was developed in which the assumed models for the tornado wind field and the drag coefficients are specified by specialized subroutines (details of such models are given in subsequent sections).

In Eq. 3 the components of the missile velocity $\bar{v}_M$ and of the wind velocity $\bar{v}_w$ must be referred to an absolute frame. The wind velocity $\bar{v}_w$ is usually specified as a sum of two parts. The first part represents the wind velocity of a stationary tornado vortex and is referred to a cylindrical system of coordinates. The second part represents the translation velocity of the tornado vortex--or, equivalently, of the cylindrical system of coordinates--with respect to an absolute frame of reference. The transformations required to represent $\bar{v}_w$ in an absolute frame are derived in Appendix A and are incorporated in the computer program. Documentation on the computer program and a sample input and output are given in Appendix C. The program, which is written in ANSI Fortran language, may be obtained on tape from the National Technical Information Service, Springfield, Virginia 22151.

For the particular case of a parallel flow, an analytical solution to the problem of the missile trajectory can be easily obtained. This solution, which can be found in Appendix B, has been used to test the program, in which the subroutine describing the wind field was modified to represent a parallel flow.

4. NUMERICAL COMPUTATIONS

It was previously noted that, for a given flow field and for given initial conditions, the missile motion depends only upon the value of the parameter C_DA/m. In this section, numerical results will be presented which show the effect of this parameter on the maximum horizontal missile speed. The calculations are based on the following assumptions:

1. The parameter C_DA/m is constant during the missile flight.
2. The tornado wind field may be represented by a vortex translating with a uniform velocity v_T along an axis denoted by Ox (Fig. 2). Let v_R, v_θ, v_z and $v_{rot} = (v_R^2 + v_\theta^2)^{1/2}$ denote the radial, tangential, vertical and horizontal velocity in the vortex flow, respectively. It is assumed that

$$v_{rot} = \frac{r}{R_m} v_m \qquad 0 \le r \le R_m \tag{5}$$

$$v_{rot} = \frac{R_m}{r} v_m \qquad R_m \le r \le \infty \tag{6}$$

where v_m is the maximum horizontal velocity in the vortex flow, r is the radius (distance from the vortex center) and R_m is the value of r at which this velocity is attained. Eqs. 5 and 6 are similar to descriptions of the flow proposed in Ref. 10 (p. 135). Furthermore, it is assumed that $v_R = 1/2\ v_\theta$ and $v_z = 0.67\ v_\theta$, i.e., (see Fig. 2).

$$v_R = - \frac{1}{\sqrt{5}} v_{rot} \tag{7}$$

$$v_\theta = \frac{2}{\sqrt{5}} v_{rot} \tag{8}$$

$$v_z = \frac{4}{3\sqrt{5}} v_{rot} \tag{9}$$

The type of model just described is referred to as the Rankine vortex and appears to be a reasonable representation of tornado flows. Estimates based on field observations suggest that it is reasonable to assume--as is done in this model--that R_m is independent of height [Ref. 10, p. 131].

The following values for the tornado wind field parameters were used in the calculations:

Table 1 - Values of v_T, v_m and R_m Used in Numerical Calculations

Tornado Type	v_T		v_m		$v_{torn} = v_m + v_T$		R_m	
	m/s*	mph	m/s*	mph	m/s*	mph	m*	ft
1	31	70	130	290	161	360	46	150
2	27	60	107	240	134	300	46	150
3	22	50	85	190	107	240	46	150
4	31	70	146	325	177	395	46	150
5	27	60	120	288	147	348	46	150
6	22	50	95	213	117	263	46	150

* Approximately

The values given in Table 1 for tornado types 1, 2, 3 are suggested in Ref. 11 as providing an acceptably low level of failure if used in the design of nuclear power plants. The values for tornado type 4, 5 and 6 were included for the purpose of studying the effect upon missile velocity of relatively small increments in the value of v_m.

3. The assumed initial conditions are: $x(0) = R_m$, $y(0) = 0$, $z(0) = 40$m, $v_{Mx}(0) = 0$, $v_{My}(0) = 0$, $v_{Mz}(0) = 0$ at time $t = 0$, where x, y, z are the coordinates of the missile (i.e., of its center of mass) and v_{Mx}, v_{My}, v_{Mz} are the missile velocity components along the x, y, z axes (Fig. 2). Also, at time $t = 0$ the center of the tornado vortex coincides with the origin 0 of the coordinate axes. The effect of assuming initial conditions different from those indicated is examined in the next section of this report.

The dependence upon the parameter C_DA/m of the maximum horizontal missile speed calculated in accordance with assumptions 1 through 3 is represented in Fig. 3 for tornado types 1 through 6 as defined in Table 1. In Fig. 3 $v_{torn} = v_T + v_m$ (Table 1).

5. SENSITIVITY OF CALCULATED RESULTS TO CHANGES IN THE ASSUMED INITIAL CONDITIONS OR IN THE ASSUMED TORNADO WIND SPEED MODEL

5.1 Changes in the Assumed Initial Position of the Missile.

For flows with $v_m = 146$ m/s, $R_m = 46$ m, and $v_T = 31$ m/s, the maximum horizontal speeds of missiles with $C_DA/m = 0.001$ and $C_DA/m = 0.01$ were calculated using the initial positions shown in Table 2. Except for the initial positions, the results of Table 2 are based on the assumptions described in the preceding section. It is seen from Table 2 that

Table 2 - Maximum Horizontal Missile Speeds, v_H^{max}, (m/s) Corresponding to Various Initial Positions

Initial Position				C_DA/m	
(1)	(2)	(3)	(4)	(5)	(6)
		x(0)	y(0)	0.001	0.01
		(meters)			
(a)	→	46	0	10	65
(b)	→	23	0	18	93
(c)	→	69	0	9	48
(d)	→	-46	0	18	82
(e)	→	0	46	16	68
(f)	→	0	23	20	84
(g)	→	0	-23	35	50
(h)	→	0	-46	54	70

1. Arrows in column (2) represent direction of translation velocity v_T.
2. Assumed elevation of missile at time t = 0: z(0) = 40m in all cases.

Table 3 - Maximum Horizontal Missile Speeds, v_H^{max}, (m/s) Corresponding to Initial Elevations z(0) = 10m, z(0) = 20m and z(0) = 40m (C_DA/m = 0.001)

Initial Position				z (0)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		x(0)	y(0)	10m	20m	40m
		(meters)				
(a)	→	46	0	6	7	10
(b)	→	23	0	2	8	18
(c)	→	69	0	9	9	9
(d)	→	-46	0	14	16	18
(e)	→	0	46	3	9	16
(f)	→	0	23	12	16	20
(g)	→	0	-23	19	29	35
(h)	→	0	-46	33	42	54

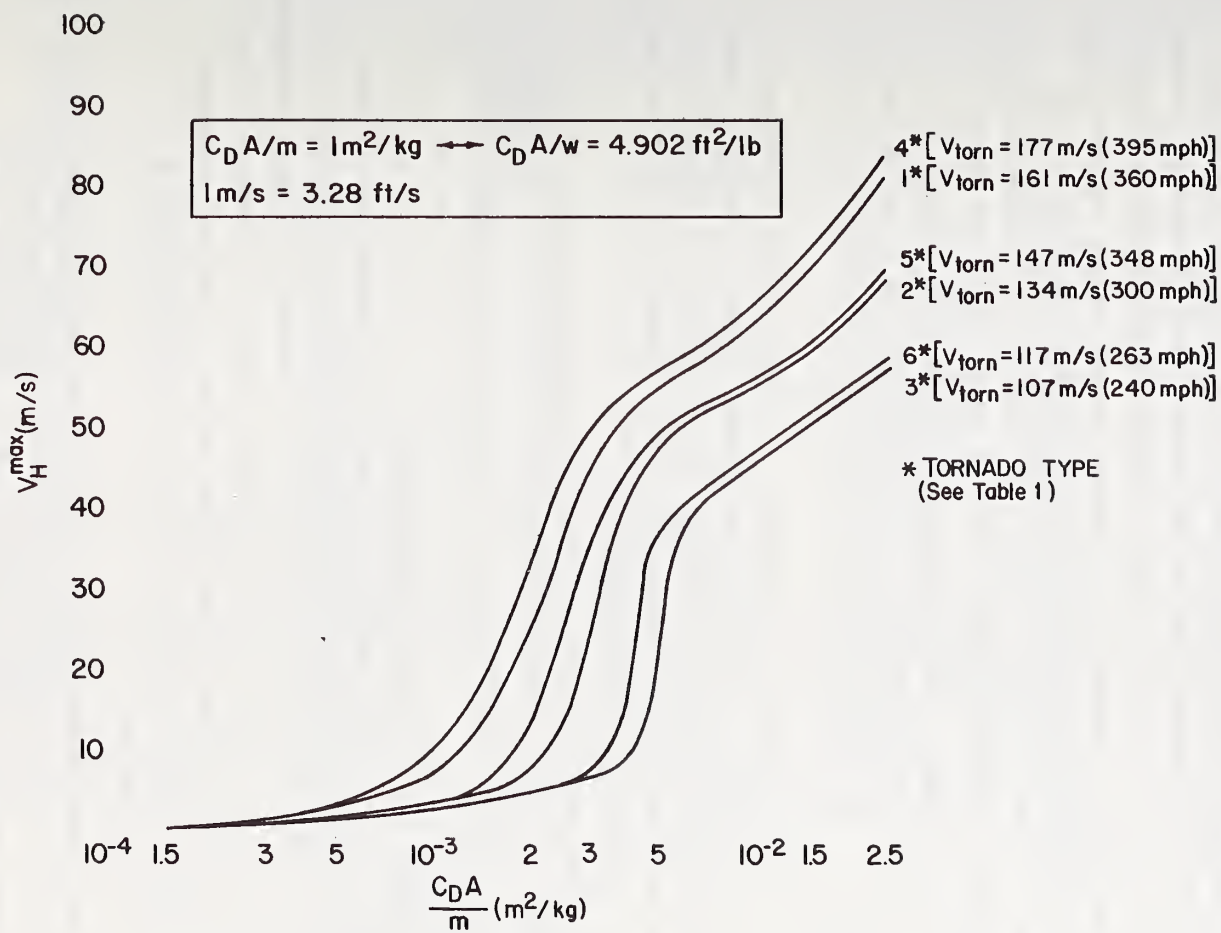

Figure 3 Variation of Maximum Horizontal Missile Speed as a Function of C_DA/m for Various Types of Tornadoes

the initial position used in the calculations of the preceding section, (position (a) of Table 2) does not result in the largest possible maximum horizontal missile speeds. It is also noted that the initial positions to which there correspond the largest possible missile speeds depend upon the value of C_DA/m. For example, for $C_DA/m = 0.01$, that initial position is $(\overline{b})$; for $C_DA/m = 0.001$, it is (h) [see Table 2].

As indicated in Note 2 of Table 2, the initial elevation assumed in the calculations was $z(0) = 40m$. If the weight of the missile is smaller than the upward drag induced by the vertical wind velocity component, then the calculated missile velocities at time t are independent of z(0). (This is the case because, in the assumed tornado model, the flow field is invariant with z). However, if the missile weight exceeds the upward drag, i.e., if the missile moves downward,--as in the case of column (5) of Table 2-- the interval between time t = 0 and the time at which the missile hits the ground decreases as z(0) decreases. Therefore, the maximum missile speed may decrease if lower values of the initial elevation z(0) are assumed. Table 3 lists speeds calculated using the assumptions z(0) = 10m and z(0) = 20m, all other values of the various parameters being the same as for column (5) of Table 2. For comparison the calculated speeds in column (5) of Table 2 were also included in Table 3.

Missile speeds were also calculated corresponding to various initial elevations $0 < z(0) < 10m$. It is of interest to note that the maximum horizontal speed of a missile with $C_DA/m = 0.001$ starting the motion from position (h) is relatively high even for low values of the initial elevation z(0). For example, for z(0) = 3m and z(0) = 5m, the maximum missile speeds are 23 m/s and 27 m/s, respectively.

Calculations were also carried out for the horizontal distances traveled by the missiles. For example, for $C_DA/M = 0.001$ and the initial position (h), the horizontal distances corresponding to z(0) = 3m, 5m, 10m, 20m, and 40m are, approximately, 20m, 30m, 50m, 90m and 160m, respectively.

5.2 Changes in the Assumed Initial Velocity of the Missile.

The results given in the preceding sections are based on the assumption that the initial velocity of the missiles is zero. If the missile is injected in the flow, for example by an explosion, this assumption is no longer valid. However, all other conditions being equal, a non-zero intial velocity does not necessarily result in a maximum missile velocity higher than that corresponding to zero initial velocity. This is illustrated in Table 4, in which type 1 tornado (see Table 1), and the conditions $v_{Mx}(0) \neq 0$, $v_{My}(0) = 0$, $v_{Mz}(0) = 0$ were assumed.

Table 4 - Maximum Horizontal Missile Speeds, v_H^{max}, (m/s) Corresponding to Various Initial Velocities

				$C_DA/m = 0.001$			$C_DA/m = 0.01$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		x(0)	y(0)	$v_{Mx}(0)$ (meters/second)					
		(meters)		0	10	20	0	10	20
(a)		46	0	8	9	20	62	58	53
(g)		0	-23	35	45	35	63	59	59

5.3 Changes in the Assumed Tornado Wind Speed Model.

5.3.1 Tornado Described by Eqs. 5 through 9 with $v_T = 0$.

Table 5 lists maximum horizontal missile speeds for the same parameter values and initial conditions as those used in Table 2, except that the translation velocity of the tornado vortex is zero, rather than 31 m/s. It is seen by comparing Tables 2 and 5 that the calculated speeds are, on the average, higher in the case $v_T = 31$ m/s, although for some initial positions the reverse is true.

Table 5 - Maximum Horizontal Missile Speeds, v_H^{max}, Corresponding to Various Initial Positions ($v_T = 0$)

Initial Position				C_DA/m	
(1)	(2)	x(0) (meters)	y(0)	(2) 0.001	(3) 0.01
(a)		46	0	30	65
(b)		23	0	7	61
(c)		69	0	19	60
(d)		-46	0	30	65
(e)		0	46	30	65
(f)		0	23	8	61
(g)		0	-23	8	61
(h)		0	-46	30	65

5.3.2 Tornado Vortex Models Based on Data Obtained During the Dallas Tornado of April 2, 1957 [13].

In this family of models, the velocities in the tornado vortex depend upon the quantities R_m = radius of maximum tangential velocity at elevation z = 0, $v_{\theta m}$ = maximum tangential velocity and R*(z) = radius beyond which the radial and vertical velocity components vanish. In addition, the flow depends on three parameters k_1, k_2, k_3, as will be shown subsequently. The radius of maximum tangential velocity at elevation z is denoted by R_m (z) and is assumed to be

$$R_m (z) = R_m + k_1 z \qquad 0 < z < 60\text{m} \qquad (10)$$

$$R_m (z) = R_m + 60\, k_1 \qquad z \geq 60\text{m} \qquad (11)$$

It is seen that the parameter k_1 is a measure of the extent to which the tornado vortex deviates from a cylindrical shape. If $k_1 = 0$ the radius of maximum tangential velocity is invariant with height, as in the model described by Eqs. 5 through 9. An assumption commonly employed in tornado-borne missile speed investigations is $k_1 = 0.45$ [see, for example, Refs. 3 and 12]. It is further assumed [3, 12]:

$$R^* (z) = C R_m (z) \tag{12}$$

where C is a coefficient which depends upon $V_{\theta m}$. In this work, it was assumed C = 2.35 for $v_{\theta m}$ = 130 m/s, C = 2.10 for $v_{\theta m}$ = 107 m/s and C = 1.80 for $v_{\theta m}$ = 85 m/s. These values are similar to those used in Ref. 3.

Let r denote the distance to the center of the tornado vortex. The components of the velocity in the tornado vortex are assumed to be

<u>0 < z < 60m</u>

$$v_R = - k_2 \frac{R^* (z) - r}{R^* (z) - R_m (z)} r \qquad r \leq R^* (z) \tag{13}$$

$$v_R = 0 \qquad r > R^*(z) \tag{14}$$

$$v_\theta = \frac{r}{R_m (z)} v_{\theta m} \qquad r \leq R_m (z) \tag{15}$$

$$v_\theta = \frac{R_m (z)}{r} v_{\theta m} \qquad r > R_m (z) \tag{16}$$

$$v_z = k_3 \frac{R^* (z) - r}{R^* (z) - R_m (z)} z + \frac{v_\theta}{3} \qquad r < R^* (z) \tag{17}$$

$$v_z = 0 \qquad r \geq R^* (z) \tag{18}$$

<u>60 m < z < 240m</u>

$$v_R = - k_2 \frac{R^* (z) - r}{R^* (z) - R_m (z)} r \frac{240 - z}{180} \qquad r \leq R^* (z) \tag{19}$$

$$v_R = 0 \qquad r > R^* (z) \tag{20}$$

$$v_\theta = \frac{r}{R_m (z)} v_{\theta m} \qquad r \leq Rm (z) \tag{21}$$

$$v_\theta = \frac{R_m (z)}{r} v_{\theta m} \qquad r > R_m (z) \tag{22}$$

$$v_z = (1.33 - \frac{z}{180}) (k_3 \frac{R^* (z) - r}{R^* (z) - R (z)} 60 + \frac{v_\theta}{3}) \qquad r < R^* (z) \tag{23}$$

$$v_z = 0 \qquad r \geq R^* (z) \tag{24}$$

Table 6 - Maximum Horizontal Missile Speeds v_H^{max}, (m/s) Corresponding to Various Parameter Values

Case	$v_{\theta m}$ (m/s)	v_T (m/s)	k_1	k_2	k_3	$C_D A/m$ m^2/kg	v_H^{max} (m/s)
1	85	22	.45	1	1	0.025	90*
2	85	22	.45	1	0	0.025	83*
3	85	22	0	1	0	0.025	77
4	85	22	0	1	1	0.025	79
5	130	31	.45	1	1	0.025	90*
6	130	31	.45	1	0	0.025	126*
7	130	31	0	1	0	0.025	80*
8	130	31	0	1	1	0.025	75*
9	130	31	.45	1	1	0.01	91*
10	130	31	.45	1	0	0.01	87
11	130	31	.25	1	0	0.01	73
12	130	31	0	1	1	0.01	60
13	130	31	0	1	0	0.01	60
14	130	31	.45	0	1	0.025	60
15	130	31	.45	0	1	0.01	36

* Calculated speeds are higher at $z > 60m$.

The most commonly used value of the parameter k_2 is unity (see, for example, Ref. 3 and 12). However, it is by no means certain that Eqs. 13 and 19 with $k_2 = 1$ are a satisfactory model of the radial velocity v_R. Indeed, according to Ref. 10 (p. 135), v_R is negligible throughout the tornado flow except in the vicinity of $r = R_m(z)$. It is, therefore, of interest to examine the influence upon the calculated missile speeds of the assumption $v_R = 0$. This can be done by assuming $k_2 = 0$ in Eqs. 13 and 19.

Similarly, while it is commonly assumed $k_3 = 1$, it is of interest to examine the case in which the value of the vertical velocity component v_z is smaller than that given by Eqs. 17 and 23 with $k_3 = 1$. This can be done, for example, by assuming $k_3 = 0$.

Calculations of missile trajectories and speeds based on the assumption that the tornado vortex is described by Eqs. 13 through 24 were carried out for the 15 cases listed in Table 6. In all cases of Table 6 it was assumed $x(0) = R_m = 46\text{m}$, $y(0) = 0$, $z(0) = 0$. Table 6 only includes the maximum calculated horizontal missile speeds v_H^{max} at elevations $z \leq 60$ m.

A discussion will now be presented of the results of Table 6. The cases corresponding to the set of parameters $k_1 = 0.45$, $k_2 = 1$, $k_3 = 1$ which, as previously indicated, are most commonly assumed in missile speed calculations, will be examined first. It is noted that the missile speeds for cases 1, 5 and 9 of Table 6 are higher than the corresponding speeds of Fig. 3, as shown in Table 7.

Table 7 - Comparison between Calculated Speeds Based on Eqs. 13 through 24 and on Eqs. 5 through 9

	Table 6		Figure 3	
$\frac{C_D A}{m}$	Case	v_H^{max} (m/s)	Tornado Type	v_H^{max} (m/s)
0.025	1	90	3*	57
0.025	5	90	1*	81
0.01	9	91	1*	63

* See Table 1

This can be explained as follows. Assume that $v_R = v_T = 0$ and that the missile starts from an initial position $x(0) = R_m$. As the missile gains momentum, it tends to fly in a straight line along the direction of the tangential velocity, i.e., at larger distances from the center of the vortex. In the case of a tornado with a radius of maximum tangential speed that is constant with height, as the missile moves away from the intial position, it will be subjected to winds of lower intensity and its speed will increase more slowly. On the other hand, if the radius of maximum tangential speed increases with height, and if the elevation of the missile increases under the action of vertical wind speed components larger than the missile weight, then as the missile moves away from the center and up it continues to fly in zones of maximum winds and thus continues to gain momentum at a fast

rate. This mechanism is modified only slightly in most, although not in all cases, if non-zero radial and translation velocities are present. The explanation just advanced is confirmed by comparing cases 4 to 1, 8 to 5, and 12 to 9. In cases 4, 8 and 12, $k_1 = 0$, i.e., the radius of maximum tangential velocity is constant, and the missile speeds are lower than in cases 1, 5, 9, in which that radius increases with height. A similar conclusion is reached by comparing cases 3 to 2, 7 to 6, 11 and 13 to 10.

The parameter k_3, which controls the magnitude of the vertical wind velocity component, does not appear to affect the missile speeds in a uniform fashion, i.e., to the value $k_3 = 0$ there correspond in certain cases lower values of the missile speed than to the value $k_3 = 1$ (e.g., case 2 vs. case 1); in other cases, the reverse is true (e.g., case 6 vs. case 5).

Some meteorologists have expressed the view that the radial velocity component v_R is smaller over most of the tornado field than that described by either Eq. 7 or Eqs. 13 and 19 with $k_2 = 1$[10]. It is therefore of interest to examine the case $k_2 = 0$. It is seen, by comparing in Table 6 cases 14 and 15 with cases 5 and 9, respectively, that the assumption $k_2 = 0$ results in considerably lower missile speeds than the assumption $k_2 = 1$. This is the case because, if $k_2 = 0$, (i.e., if $v_R = 0$), the missile is ejected (i.e., it flies at larger distances from the center of the vortex and therefore, in a region of lower speeds) sooner than if $k_2 = 1$, (i.e., than if a radial velocity is present that resists the tendency of the missile to fly away from the center). This reasoning is valid for the assumed initial condition $x(0) = R_m$, $y(0) = 0$. It is conceivable however, that initial conditions exist which might result in higher missile speeds for $k_2 = 0$ than for $k_2 = 1$.

6. AERODYNAMIC FORCES

In this work it was assumed that the aerodynamic force acting upon a missile is

$$\overline{D} = 1/2\ \rho\ C_D A\ \left| \overline{v}_w - \overline{v}_M \right| (\overline{v}_w - \overline{v}_M) \tag{24}$$

where ρ = air density, $\overline{v}_w$ = wind velocity, $\overline{v}_M$ = missile velocity and $C_D A$ is a suitably chosen quantity with the dimensions of an area. As indicated in Section 2, Eq. 2 may be assumed to be valid under random tumbling conditions, i.e., if the motion is such that the body "presents all possible aspects to the flow,. . . the orientation of the surface elements with respect to the flow sweeps through all the possible angles" (Ref. 14) and, moreover, the angular speed of the tumbling body is sufficiently large. In this connection it may be argued that, instantaneously, lift forces do exist, and that the momentum they impart to the missile may not be negligible if the rotational motion of the missile is relatively slow. The acceleration of the missile would then no longer be parallel, at every instant, to the vector $\overline{v}_w - \overline{v}_M$ and the missile trajectory would deviate from that predicted by using the aerodynamic model implicit in Eq. 2. It is believed, however, that the effect of such deviations on the maximum speed of the tornado-borne missile is

comparable to the effect of changes in the initial conditions of the problem such as were studied in Section 5. For the purposes of this report, the existence of lift forces which are not taken into account in the calculations is believed not to invalidate the aerodynamic model used herein.

The value $C_D A$ that corresponds to tumbling conditions can, in principle, be determined experimentally. Unfortunately, little information on this topic appears to be presently available. Ref. 14 examines tumbling motions for supersonic wind conditions, while Ref. 15 contains information on tumbling under flow conditions corresponding to Mach numbers 0.5 to 3.5. Hoerner extrapolated the data of Ref. 15 to lower subsonic speeds (Ref. 16, p. 14-16, Fig. 7); according to this extrapolation, for a randomly tumbling cube the quantity $C_D A$ equals, approximately, the average of the products of the projected areas corresponding to "all positions statistically possible" by the respective static drag coefficients (Ref. 16, p. 14-16 and P. 13-17). An investigation into this question is currently carried out, within the framework of this project, by Colorado State University (CSU). The theoretical and experimental results of this investigation will be reported in a separate document by CSU.

In the absence of more experimental information, it appears reasonable to assume that the effective product $C_D A$ is given by the expression

$$C_D A = c\,(C_{D_1} A_1 + C_{D_2} A_2 + C_{D_3} A_3) \tag{25}$$

in which $C_{D_i} A_i$ $(i = 1, 2, 3)$ are products of the projected areas corresponding to the cases in which the principal axes of the body are parallel to the vector $\bar{v}_W - \bar{v}_M$, by the respective static drag coefficients. In Eq. 25, c is a coefficient assumed to be 0.50 for planks, rods, pipe and poles and 0.33 for the automobile.

An upper bound for the quantity $C_D A$ is believed to be

$$(C_D A)_{u.b.} = C_{D_1} A_1 \tag{26}$$

in which $C_{D_1} A_1$ is the largest of the quantities $C_{D_i} A_i$ $(i = 1, 2, 3)$.

The Reynolds number is defined as

$$Re = \frac{Vd}{\nu}$$

where V = fluid velocity relative to the body, d = characteristic dimension of the body (in the case of a cylinder, d = diameter) and ν = kinematic viscosity ($\nu \simeq 1.5 \times 10^{-5}$ m/s for air). For a circular cylinder $Re \simeq .67 \times 10^5$ Vd, where V and d are expressed in m/s and m, respectively. For $Re > 4 \times 10^5$, i.e., the Reynolds number is in the supercritical range and it may therefore be assumed, conservatively, $C_{D_1} \simeq 0.7$ (see Ref. 8, p. 67). In the case of the 1 inch (2.54 cm) rod, it may be assumed that Re is in the subcritical range even for velocities V of the order of 100 m/s and, therefore, that $C_{D_1} \simeq 1.2$ (Ref. 8).

7. SPEEDS OF SELECTED POTENTIAL TORNADO-BORNE MISSILES

In this section calculated speeds of selected potential tornado-borne missiles will be given, based on the following assumptions:

(1) The model of the tornado vortex consisting of Eqs. 5 through 9 is valid, with the parameter values corresponding to cases 1, 2 and 3 of Table 1.

(2) The initial conditions are $x(0) = R_m$, $y(0) = 0$, $z(0) = 40m$ (for comments on the initial condition $z(0) = 40m$, see p. 10 of this report); $v_{Mx}(0) = v_{My}(0) = v_{Mz}(0) = 0$. Assumptions (1) and (2) just described were used in calculating the values on curves 1, 2 and 3 of Fig. 3.

(3) The effective product C_DA is given by Eq. 25.

The results of the calculations based on these assumptions are shown in Table 8.

The missile speeds of Table 8 are based on a set of assumptions which, while reasonable, might in certain cases not correctly reflect the actual physical phenomenon. It follows from Sections 5 and 6 that the order of magnitude of uncertainties in the estimates of maximum missile speeds can in certain cases be as high as 50% or even more. Whether or not actual missile speeds will be higher than those listed in Table 8 depends in large measure on the extent to which the tornado flow model consisting of Eqs. 5 through 9 (the so-called Rankine vortex) is realistic. In particular, if, as suggested in Ref. 10, the radial and vertical velocity components in a tornado are actually lower than those given by this model, it could be expected--all other conditions being equal--that the predictions of Table 8 are conservative. If, on the other hand, the actual tornado flow is more closely represented by Eqs. 13 through 24 with certain unfavorable values of the parameters included in these equations, then higher missile speeds than those of Table 8 may occur, as shown in Tables 6 and 7.

The speeds of Table 8 may also be exceeded if unfavorable initial conditions obtain. The uncertainties pertaining to the tornado flow modeling are due to the lack of reliable information; on the other hand, those pertaining to the initial conditions are a consequence of the probabilistic nature of the problem. Probabilities of occurrence may be assigned to each set of initial conditions. The probability that (a) the wind speed will reach the intensity levels of Table 1, (b) that a missile starts from a highly unfavorable set of initial conditions and (c) that it will hit a certain installation with a speed V_H^{max} can be expected to be negligibly low. Such probabilities can in principle be evaluated using, for example, procedures similar to those outlined in Ref. 18. Attempts to calculate such probabilities are beyond the scope of this work.

Table 8 - Characteristics and Maximum Horizontal Speeds of Selected Missiles

		Dimensions	Weight (lb/ft)	Mass (kg/m)	C_{D_1}	C_{D_2}	C_{D_3}	$C_D A/w$ (ft^2/lb)	$C_D A/m$ (m^2/kg)	V_H^{max} Tornado Type 1[a]	Tornado Type 2[b]	Tornado Type 3[c]
1	Wooden Plank	3 5/8" x 11 3/8" x 12' (0.092m x 0.289m x 3.66m)	8.2 to 11[d] (say, 9.6)	12.2 to 16.3 (say 14.3)	2.0	2.0	2.0	0.132	0.0270	272fps (83 m/s)	230fps (70 m/s)	190fps (58 m/s)
2	6" Sch. 40 Pipe	6.625" (diam) x 15' (length) (0.168m x 4.58m)	18.97	28.18	0.7	2.0		0.0212	0.0043	171fps (52 m/s)	138fps (42 m/s)	33fps (10 m/s)
3	Automobile	16.4' x 6.6' x 4.3' (5m x 2m x 1.3m)	4,000 lb (total weight)	1,810 kg (total mass)	2.0	2.0	2.0	0.0343	0.0070	193fps (59 m/s)	170fps (52 m/s)	134fps (41 m/s)
4	1" Solid Steel Rod	1" (diam) x 3' (length) (0.0254m x 0.915m)	2.67	4.0	1.2	2.0	1.2	0.0190	0.0040	167fps (51 m/s)	131fps (40 m/s)	326fps (8 m/s)
5	13.5" Utility Pole	13.5" (diam) x 35' (length) (0.343 cm x 10.68m)	27.5 to 36.5 (say, 32)	40.8 to 54.2 (say, 47.5)	0.7	2.0	0.7	0.0254	0.0052	180fps (55 m/s)	157fps (48 m/s)	85fps (26 m/s)
6	12" Sch 40 Pipe	12.75" (diam) x 15' (length) (0.32m x 4.58m)	49.56	73.6	0.7	2.0	0.7	0.016	0.00330	154fps (47 m/s)	92fps (28 m/s)	23fps (7 m/s)

[a] V_{torn} = 161 m/s (360 mph) - see Table 1.

[b] V_{torn} = 134 m/s (300 mph) - see Table 1.

[c] V_{torn} = 107 m/s (240 mph) - see Table 1.

d See Ref. 17.

The uncertainties regarding the actual aerodynamic drag coefficients constitute another source of error. It is noted that the curves in Fig. 3 are S-shaped. Large errors in the assumed value of the quantity C_DA may therefore in certain cases result in considerable changes of the estimated value of V_H^{max}. For example, if, for a 12 in Sch. 40 pipe (entry 6 in Table 8) it was assumed $C_DA/m = 0.0066$ kg/m^2 instead of $C_DA/m = 0.0033$ kg/m^2, it follows from Fig. 3 that $V_H^{max} = 40$ m/s, rather than $V_H^{max} = 7$ m/s, as in Table 8. It is interesting to note, on the other hand, that as long as a change in the assumed value of C_DA/m does not displace the point from the upper or from the lower branch of an S-shaped curve, the sensitivity of V_H^{max} to even considerable changes in the value of C_DA/m is fairly small. For example, if for the 6 in Sch. 40 pipe (entry 2 in Table 8) it is assumed that $C_DA/m = 0.0086$, then, for a tornado type 1, $V_H^{max} = 61$ m/s (see Fig. 3), whereas to the assumption $C_DA/m = 0.0043$ there corresponds $V_H^{max} = 52$ m/s, i.e., to an error of 100% in the value of C_DA/m there corresponds an error of only 17% in the estimated value of V_H^{max}.

Finally, it must be noted that the actual missiles may have properties that are more unfavorable than those listed in Table 8; for example, the case is mentioned in Ref. 19 of a beam attached, during its flight, to a portion of a carport roof which considerably increased the surface area of the missile and, therefore, the parameter C_DA/m.

8. CONCLUSIONS

In the preceding section calculated maximum speeds of tornado-borne missiles are given in Table 8, based on a set of assumptions believed to be reasonable. However, in assessing these speeds, it must be recognized that:

1. The problem of determining tornado-borne missile speeds has a probabilistic character. As shown in the body of the report, unfavorable initial conditions may obtain--to which there correspond relatively low probabilities of occurrence--for which the maximum missile speeds would be higher than in Table 8. The estimation of such probabilities is beyond the scope of the investigation covered by this report.

2. Estimates of tornado-borne missile speeds are also affected by significant uncertainties with regard to: (a) the detailed structure of the tornado flow and (b) to the aeordynamic behavior of the missile. Under certain assumptions regarding one or both of these factors, calculated missile speeds can be higher than those of Table 8. However, it is believed that the assumption used to derive the values of Table 8 are conservative. In particular, it is believed that the actual vertical wind speeds are lower than indicated by Eq. 9, so that the relatively heavy missiles would tend to hit the ground sooner than calculated on the basis of this equation, with a consequent reduction in the calculated maximum missile speed.

In spite of the many uncertainties involved, the writers believe that the assumptions used to estimate the speeds of Table 8 are sufficiently conservative for purposes of nuclear power plant design. It is the writers' judgement that, although higher values of tornado-borne missile speeds are conceivable, their probabilities of occurrence, for any given tornado strike, are low.

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APPENDIX A - EQUATIONS OF MOTION

A1.1 Absolute and Translating Frames of Reference

Consider the absolute frame of reference 0_1 xyz (Fig. A1.1) and a frame 0_2 x'y'z such that 0_2z is parallel to 0_1z. It is assumed that the frame 0_2 x'y'z translates with respect to 0_1 xyz with a constant velocity, the components of which (in the 0_1 xyz frame) are $(v_{Tx}, v_{Ty}, 0)$. The angle between $0_2x'$ and 0_1x being denoted by θ_c,

$$\cos\theta_c = \frac{v_{Tx}}{(v_{Tx}^2 + v_{Ty}^2)^{1/2}} \tag{A1.1}$$

$$\sin\theta_c = \frac{v_{Ty}}{(v_{Tx}^2 + v_{Ty}^2)^{1/2}} \tag{A1.2}$$

A1.2 Vectors of Position

Using the notations of Fig. A1.1, the vectors of position of the particle P in the absolute and in the translating frame of reference may be written as follows:

Absolute frame (with origin 0_1)

$$\bar{R} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k} = r\cos\theta\,\bar{i} + r\sin\theta\,\bar{j} + z\,\bar{k} \tag{A1.3}$$

where $\bar{i}$, $\bar{j}$, $\bar{k}$ are unit vectors along the axes 0_1x, 0_1y, 0_1z respectively.

Translating frame (with origin 0_2)

$$\bar{R}' = x'\,\bar{i}' + y'\,\bar{j}' + z\,\bar{k} = r'\cos\theta'\,\bar{i}' + r'\sin\theta'\,\bar{j}' + z\,\bar{k} \tag{A1.4}$$

where $\bar{i}'$, $\bar{j}'$, $\bar{k}$ are unit vectors along the axes $0_2x'$, $0_2y'$, 0_2z, respectively

A1.3 Transformations of Unit Vectors

The unit vectors $\bar{r}$, $\bar{\theta}$, $\bar{k}$ of the revolving frame of reference (see Fig. A1.1) may

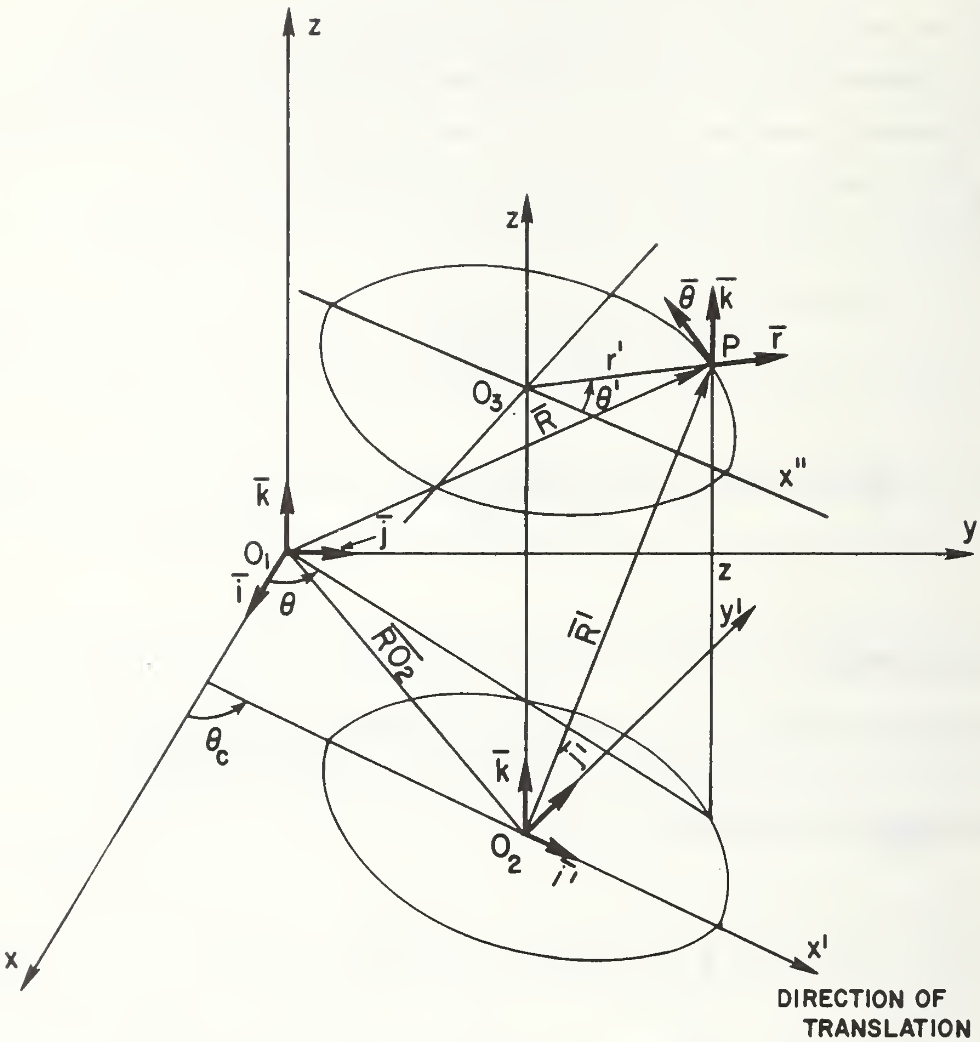


Figure A1 Notations

be written in terms of the unit vectors of the translating frame $0_2x'y'z$ and the angle between the lines 0_3P and $0_3x''$, where $0_3x''$ is parallel to $0_2x'$, as follows:

$$\begin{aligned}
\bar{r} &= \cos\theta'\ \bar{i}' + \sin\theta'\ \bar{j}' \\
\bar{\theta} &= -\sin\theta'\ \bar{i}' + \cos\theta'\ \bar{j}' \qquad \text{(A1.5)} \\
\bar{k} &= \bar{k}
\end{aligned}$$

The unit vectors $\bar{i}'$, $\bar{j}'$, $\bar{k}$, of the translating frame $0_2\ x'y'z$ may be written in terms of the unit vectors of the absolute frame of reference and of the angle θ_c as follows:

$$\begin{aligned}
\bar{i}' &= \cos\theta_c\ \bar{i} + \sin\theta_c\bar{j} \\
\bar{j}' &= -\sin\theta_c\ \bar{i} + \cos\theta_c\ \bar{j} \qquad \text{(A1.6)} \\
\bar{k} &= \bar{k}
\end{aligned}$$

A1.4 Expression of Wind Velocity in an Absolute Frame of Reference

Let the vortex wind velocity with respect to the translating frame of reference be denoted $\bar{v}_\omega$, and let its components along the vectors $\bar{r}$, $\bar{\theta}$ and $\bar{k}$ be denoted by $v_{\omega r}$, $v_{\omega\theta}$ and $v_{\omega k}$, respectively, i.e.,

$$\bar{v}_\omega = v_{\omega r}\ \bar{r} + v_{\omega\theta}\ \bar{\theta} + v_{\omega k}\ \bar{k} \qquad \text{(A1.7a)}$$

With respect to the absolute frame of reference, $\bar{v}_\omega$ may be written as

$$\bar{v}_\omega = v_{\omega x}\ \bar{i} + v_{\omega y}\ \bar{j} + v_{\omega z}\ \bar{k} \qquad \text{(A1.7b)}$$

The components $v_{\omega x}$, $v_{\omega y}$ and $v_{\omega z}$ are obtained by substituting Eqs. A1.5 and A1.6 into Eq. A1.7a, i.e., in matrix form,

$$\begin{bmatrix} v_{\omega x} \\ v_{\omega y} \\ v_{\omega z} \end{bmatrix} = \begin{bmatrix} \cos\theta_c & -\sin\theta_c & 0 \\ \sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta' & -\sin\theta' & 0 \\ \sin\theta' & \cos\theta' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{\omega r} \\ v_{\omega\theta} \\ v_{\omega k} \end{bmatrix} \tag{A1.8}$$

The total wind velocity $\bar{v}_w$ consists of the sum of the vortex wind velocity and the translating velocity of the vortex, i.e.,

$$\bar{v}_w = v_{wx}\,\bar{i} + v_{wy}\,\bar{j} + v_{wk}\,\bar{k} \tag{A1.9}$$

where

$$\left.\begin{aligned} v_{wx} &= v_{\omega x} + v_{Tx} \\ v_{wy} &= v_{\omega y} + v_{Ty} \\ v_{wz} &= v_{\omega z} \end{aligned}\right\} \tag{A1.10}$$

and $v_{\omega x}$, $v_{\omega y}$, $v_{\omega z}$ are given by Eqs. A1.8. In Eqs. A1.10 and A1.8, the quantities v_{Tx}, v_{Ty}, $v_{\omega r}$, $v_{\omega\theta}$, $v_{\omega k}$ and θ_c are specified. The quantity θ' is a function of time and is obtained from the relations

$$\left.\begin{aligned} \cos\theta' &= \frac{x'}{r'} \\ \sin\theta' &= \frac{y'}{r'} \\ r' &= (x'^2 + y'^2)^{1/2} \end{aligned}\right\} \tag{A1.11}$$

in which x' and y' are determined as explained in the following.

Let the initial conditions of the problem be

$t_o \equiv$ initial time

$(x_o, y_o, z_o) \equiv$ coordinates of particle at time t_o in the absolute frame

$(v_{Pxo}, v_{Pyo}, v_{Pzo}) \equiv$ velocity components of particle at time t_o in the absolute frame

$(x0_{2o}, y0_{2o}, 0) \equiv$ position of 0_2 (origin of the translating frame) at time t_o, in the absolute frame.

At time t, the position vector of the origin 0_2 is

$$\overline{R0}_2 = [x0_{2o} + v_{Tx} (t - t_o)]\ \overline{i} + [y0_{2o} + v_{Ty} (t - t_o)\ \overline{j}] \tag{A1.12}$$

so that for any time $t \geq t_o$ we have

$$\overline{R}' = \overline{R} - \overline{R0}_2 \tag{A1.13}$$

where $\overline{R} = x\ \overline{i} + y\ \overline{j} + z\overline{k}$, i.e.,

$$\begin{bmatrix} x_{R'} \\ y_{R'} \\ z_{R'} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} x0_{2o} + v_{Tx} (t - t_o) \\ y0_{2o} + v_{Ty} (t - t_o) \\ 0 \end{bmatrix} \tag{A1.14}$$

Since

$$x_{R'}\ \overline{i} + y_{R'}\ \overline{j} + z_{R'}\ \overline{k} = x'\ \overline{i}' + y'\ \overline{j}' + z'\ \overline{k} \tag{A1.15}$$

it follows, using the inverse of the transformation A1.6,

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{R'} \\ y_{R'} \\ z_{R'} \end{bmatrix} \tag{A1.16}$$

where x_R', y_R', z_R', are given by Eq. A1.14. The quantities x', y', z', in Eq. A1.11, and thus the quantity θ' in Eqs A1.10, A1.8 are expressed in terms of the coordinates of point P with respect to the absolute frame, and of the quantities $x0_{xo}$, $y0_{2o}$, v_{Tx}, v_{Ty}, t_o and t. The problem of expressing the components in the absolute frame of the wind velocity at point P (x, y, z) in terms of the vortex wind velocity components $v_{\omega r}$, $v_{\omega\theta}$, $v_{\omega k}$, of the translation velocity components v_{Tx}, v_{Ty}, of the initial conditions and of the coordinates x, y, z of the point P is thus solved.

A1.5 Equations of Motion

The equations of motion may be written as

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = 1/2\, \rho\, C_D A |v_{Rel}|\, \bar{v}_{Rel} - g\bar{k} \qquad (A1.17)$$

where m = mass of particle, ρ = air density, C_D = drag coefficient, A = area of the particle, $\bar{v}_{Rel} = (v_{wx} - \dot{x})\,\bar{i} + (v_{wy} - \dot{y})\,\bar{j} + (v_{wz} - \dot{z})\,\bar{k}$, and g = acceleration of gravity.

APPENDIX B - ANALYTIC SOLUTION TO THE EQUATIONS OF MOTION FOR A UNIFORM WIND FIELD.

In the case of a uniform wind field an analytic solution of the equations of motion of a particle may be obtained as shown herein. This solution was used to test the computer program, in which the wind field subroutine was suitably modified. The following assumptions were used:

1) The initial velocity of the particle is zero

2) The motion occurs in the x - z plane only

3) The wind velocity vector is at all points parallel to the horizontal axis 0_x and has the constant magnitude v_w.

The equation of motion in the horizontal direction is

$$\frac{dx^2}{dt^2} = \alpha \left(v_w - \frac{dx}{dt}\right)^2 \qquad (A2.1)$$

where

$$\alpha = 1/2 \ \frac{\rho}{m} C_D A, \text{ or}$$

$$\int \frac{d(\frac{dx}{dt})}{(v_w - \frac{dx}{dt})^2} = \int \alpha \, dt + C_1 \qquad (A2.2)$$

With the change of variables $u = \frac{dx}{dt} - v_w$, $du = d \frac{dx}{dt}$, Eq A22 becomes

$$\int \frac{du}{u^2} = \alpha \ t + C_1 \qquad (A2.3)$$

$$-\frac{1}{\frac{dx}{dt} - v_w} = \alpha \ t + C_1 \qquad (A2.4)$$

$$\frac{dx}{dt} = v_w - \frac{1}{\alpha t + C_1} \qquad (A2.5)$$

Integrating Eq. A2.5, there follows

$$x = v_w t - \int \frac{dt}{\alpha t + C_1} + C_2 \qquad (A2.6)$$

With the change of variables $\alpha \ t + C_1 = u$, $\alpha \ dt = du$, $dt = \frac{du}{\alpha}$, Eq. A2.6 becomes

$$x = v_w t - \frac{1}{\alpha} \int \frac{du}{u} + C_2 \qquad (A2.7)$$

$$x = v_w t - \frac{1}{\alpha} \ln (\alpha t + C_1) + C_2 \qquad (A2.8)$$

The constants of integration are determined as follows:

At $t = o$, $x = x_o$ and $\frac{dx}{dt} = o$, i.e.,

$$x_o = -\frac{1}{\alpha} \ln C_1 + C_2 \qquad \text{(from Eq. A2.8)} \qquad (A2.9)$$

$$0 = v_w - \frac{1}{C_1} \qquad \text{(from Eq. A2.5)} \qquad (A2.10)$$

Thus

$$C_1 = \frac{1}{v_w} \qquad (A2.11)$$

$$C_2 = x_o - \frac{1}{\alpha} \ln v_w \qquad (A2.12)$$

and

$$x = x_o + v_w t - \frac{1}{\alpha} \ln (\alpha v_w t + 1) \qquad (A2.13)$$

APPENDIX C

DOCUMENTATION,

AND

SAMPLE INPUT AND OUTPUT

OF COMPUTER PROGRAM

TORNADO*WIND(1).MAIN$EX(1)

```
C
C                             NATIONAL BUREAU OF STANDARDS
C                             APPLIED MATHEMATICS DIVISION
C
C                                       TORWD1
C
C        A FORTRAN APPLICATIONS PROGRAM TO STUDY THE MOTION OF AN OBJECT
C        UNDER THE INFLUENCE OF A TORNADO WIND FIELD.
C
C                                  MARTIN CORDES
C
C                                   APRIL 1976
C
C-----------------------------------------------------------------------
C
C  PURPOSE.
C
C      THIS FORTRAN PROGRAM INTEGRATES IN TIME THE FOLLOWING EQUATIONS OF
C      MOTION FOR A RIGID BODY ACTED ON BY A DRAG FORCE IN A
C      GRAVITATIONAL FIELD.
C
C                 D**2(PX) / DT**2 = ALPHA * VRX
C
C                 D**2(PY) / DT**2 = ALPHA * VRY
C
C                 D**2(PZ) / DT**2 = ALPHA * VRZ - G
C
C                            AND
C
C                 ALPHA = CD * A * RHO * VR / 2   .
C
C        HERE
C
C          D**2(X) / DT**2    IS THE SECOND DERIVATIVE OF X WITH RESPECT
C                             TO TIME.
C
C          PX                 ARE THE X, Y, Z COORDINATES OF THE
C          PY                 PARTICLE AS A FUNCTION OF TIME IN THE
C          PZ                 ABSOLUTE FRAME.
C
C          VRX                ARE THE X, Y, Z COMPONENTS OF THE RELATIVE
C          VRY                VELOCITY OF THE WIND WITH RESPECT TO A
C          VRZ                STATIONARY PARTICLE IN THE ABSOLUTE FRAME.
C
C          G                  IS THE MAGNITUDE OF GRAVITY.
C
C          CD                 IS THE DRAG COEFFICIENT.
C
C          A                  IS THE AREA ASSOCIATED WITH THE BODY.
C
C          RHO                IS THE DENSITY OF AIR.
C
C          VR                 IS SQRT(VRX ** 2 + VRY ** 2 + VRZ ** 2).
C
C      THIS SYSTEM OF THREE, SECOND ORDER EQUATIONS IS CONVERTED TO A
C      SYSTEM OF SIX, FIRST ORDER EQUATIONS BY A STANDARD TECHNIQUE.
C      THE EQUIVALENT FIRST ORDER SYSTEM IS INTEGRATED BY THE ODE
```

```
C      SOLVER.
C
C      THE COMPONENTS OF THE PARTICLE VELOCITY AND THE WIND VELOCITY ARE
C      WITH RESPECT TO AN ABSOLUTE FRAME. THE WIND VELOCITY IN THE
C      ABSOLUTE FRAME IS THE SUM OF THE WIND VELOCITY FROM A STATIONARY
C      TORNADO VORTEX AND THE TRANSLATION VELOCITY OF THE TORNADO VORTEX.
C
C      INITIAL CONDITIONS.
C
C         AT TIME = TO THE X, Y, Z COORDINATES AND VELOCITY COMPONENTS
C         OF THE PARTICLE ARE SPECIFIED. IN ADDITION, THE X, Y
C         COORDINATES OF THE ORIGIN OF THE TRANSLATING FRAME WHICH THE
C         TORNADO IS STATIONARY IN ARE SPECIFIED.
C
C      CONDITIONS INDEPENDENT OF TIME.
C
C         THE X, Y VELOCITY COMPONENTS OF THE TRANSLATING FRAME ARE
C         CONSTANT.
C
C      FOR FURTHER DETAILS CONSULT REFERENCE 2).
C
C   APPLICABILITY AND RESTRICTIONS.
C
C      THIS PROGRAM IS INTENDED FOR WORK THAT REQUIRES MODERATE ACCURACY
C      AND MODERATE TIME PERIODS OF INTEGRATION.
C
C      THE CURRENT VERSION PRODUCES PLOTS OF AT MOST 101 DATA POINTS. THE
C      DATA IS FROM THE TABULAR RESULTS GENERATED AT THE FIRST 101 PRINT
C      STEPS. TO PREVENT A TRUNCATED PLOT FROM BEING GENERATED BE SURE
C      THAT TIMEIT AND FXTMIT SATISFY TIMEIT .LT. 100 * FXTMIT. HERE
C      TIMEIT IS THE TIME INTERVAL OF INTEGRATION AND FXTMIT IS THE TIME
C      INTERVAL BETWEEN SUCCESSIVE PRINT STEPS.
C
C   PROGRAM STRUCTURE.
C
C      SUBPROGRAM DIRECTORY.
C
C         COMMENT  ADDITIONAL SUBPROGRAMS OTHER THAN FORTRAN INSTRINSIC
C                  FUNCTIONS AND FORTRAN BASIC EXTERNAL FUNCTIONS
C                  NECESSARY FOR THE EXECUTION OF THIS MAIN PROGRAM ARE
C                  LISTED BELOW IN ALPHABETICAL ORDER. EACH IS
C                  ACCOMPANIED BY A SHORT FUNCTIONAL DESCRIPTION.
C
C         DFUN     THIS SUBROUTINE COMPUTES THE RIGHT HAND SIDE OF THE
C                  FIRST ORDER ODE SYSTEM. DY IS THE LOCAL NAME OF DFUN
C                  IN VOADAM.
C
C         DRAG     (USER SUPPLIED.) THIS SUBROUTINE COMPUTES THE DRAG
C                  COEFFICIENT.
C
C         INPUT    THIS SUBROUTINE READS IN OR CONTROLS THE READING IN OF
C                  ALL RELEVANT DATA AND PARAMETERS EXCEPT THE PARAMETERS
C                  REQUIRED BY THE ODE SOLVER AND WHICH AFFECT OUTPUT.
C
C         OUTPUT   THIS SUBROUTINE READS IN PARAMETERS THAT CONTROL WHICH
C                  PLOTS ARE TO BE PRODUCED AND SUPERVISES THE PRODUCTION
C                  AF ALL OUTPUT.
C
```

```
C        PLOT      THIS SUBROUTINE PRODUCES A ONE PAGE PRINTER PLOT.
C
C        TORWDF    (USER SUPPLIED.) THIS SUBROUTINE COMPUTES THE RADIAL,
C                  ANGULAR, AND Z COMPONENTS OF THE WIND AT THE PARTICLE
C                  AS MEASURED IN THE TRANSLATING FRAME.
C
C        TRIGFS    THIS SUBROUTINE COMPUTES THE COSINE AND SINE OF THE
C                  BASE ANGLE OF A RIGHT TRIANGLE WITH BASE AND VERTICAL
C                  SIDE GIVEN.
C
C        VOADAM    THIS SUBROUTINE IS THE ODE SOLVER. EACH CALL DOES ONE
C                  INTEGRATION STEP.
C
C     EXECUTION TREE.
C
C        COMMENT   THE EXECUTION TREE IS A STRUCTURE THAT DESCRIBES THE
C                  SUBPROGRAM INTERACTION DURING EXECUTION. THE TREE IS
C                  COMPOSED OF A NUMBER OF LEVELS. EACH LEVEL IS COMPOSED
C                  OF ONE OR MORE BLOCKS. A BLOCK IS COMPOSED OF A
C                  CONTIGUOUS SET OF TREE ELEMENTS. EACH TREE ELEMENT
C                  HAS THE FORM
C                                NUMB1(NAME, NUMB2)
C                  WHERE
C
C                     NUMB1      IS THE NUMBER OF THE ELEMENT IN THE
C                                CURRENT LEVEL.
C
C                     NAME       IS THE NAME OF THE SUBPROGRAM ASSOCIATED
C                                WITH THAT ELEMENT.
C
C                     NUMB2      IS THE NUMBER OF THE FIRST ELEMENT OF
C                                THE BLOCK AT THE NEXT GREATER LEVEL WHICH
C                                CONTAINS ALL SUBPROGRAMS CALLED BY
C                                SUBPROGRAM NAME. IF NUMB2 IS ZERO THEN
C                                SUBPROGRAM NAME CALLS NO SUBPROGRAMS.
C
C           LEVEL 0          LEVEL 1          LEVEL 2          LEVEL 3
C
C        1(MAIN  , 1)    1(INPUT , 1)    1(TORWDF, 0)    1(TORWDF, 0)
C                        2(VOADAM, 4)    2(DRAG  , 0)    2(DRAG  , 0)
C                        3(OUTPUT, 5)    3(TRIGFS, 0)    3(TRIGFS, 0)
C
C                                        4(DFUN  , 1)
C
C                                        5(TORWDF, 0)
C                                        6(DRAG  , 0)
C                                        7(TRIGFS, 0)
C                                        8(PLOT  , 0)
C
C     PROGRAM I/O.
C
C        INPUT.
C
C           ROUTINE.    LOGICAL UNIT(S).     EXECUTION TREE ELEMENT.
C
C           MAIN               5             LEVEL  0,   ELEMENT  1
C
C           INPUT              5             LEVEL  1,   ELEMENT  1
```

```
C
C           OUTPUT                  5                 LEVEL  1.   ELEMENT  3
C
C           TORWDF                  5                 LEVEL  2.   ELEMENT  1
C
C           DRAG                    5                 LEVEL  2.   ELEMENT  2
C
C        OUTPUT.
C
C           ROUTINE.    LOGICAL UNIT(S).      EXECUTION TREE ELEMENT.
C
C           MAIN                    6                 LEVEL  0.   ELEMENT  1
C
C           OUTPUT                  6                 LEVEL  1.   ELEMENT  3
C
C           TORWDF                  6                 LEVEL  2.   ELEMENT  5
C
C           DRAG                    6                 LEVEL  2.   ELEMENT  6
C
C           PLOT                    6                 LEVEL  2.   ELEMENT  8
C
C  PROGRAM USAGE (INPUT).
C
C     DATA GROUPINGS.
C
C        GROUP 1.
C
C           SUBGROUP 1.1.  TORNADO WIND FIELD PARAMETER CARD(S).
C
C              CARD.               VARIABLE LIST.               FORMAT.
C
C                 SEE SUBROUTINE TORWDF FOR THE EXACT SET OF PARAMETERS
C                 TO READ IN AND THEIR FORMAT.
C
C           SUBGROUP 1.2.  DRAG COEFFICIENT PARAMETER CARD(S).
C
C                 SEE SUBROUTINE DRAG FOR THE EXACT SET OF PARAMETERS TO
C                 READ IN AND THEIR FORMAT.
C
C        GROUP 2.
C
C           SUBGROUP 2.1.  PARTICLE PARAMETER CARD.
C
C              CARD.               VARIABLE LIST.              FORMAT.
C
C              1         AREA, MASS                            2E12.0
C
C           VARIABLE DEFINITION TABLE (GROUP 2).
C
C              AREA      ARE THE AREA, MASS OF THE PARTICLE.
C              MASS
C
C        GROUP 3.
C
C           SUBGROUP 3.1.  INITIAL CONDITION CARDS.
C
C              CARD.               VARIABLE LIST.              FORMAT.
C
```

```
C              1          XPO, YPO, ZPO, VXPO, VYPO, VZPO      6E12.0
C
C              2          XOTF, YOTF, VXTF, VYTF, TO           5E12.0
C
C          SUBGROUP 3.2.  FINAL CONDITION CARD.
C
C              3          TIMEIT                               E12.0
C
C          SUBGROUP 3.3.  ODE SOLVER PARAMETER CARD.
C
C              4          HI, FXTMIT, EPSI                     3E12.0
C
C          SUBGROUP 3.4.  PLOT AND VALIDATION PRAMETER CARD.
C
C              5          WTPTZY, WTPTYX, WTPTZX, WTPTXT,      7I2
C                         WTPTSX, WTPTHT, WTVALD
C
C          VARIABLE DEFINITION TABLE (GROUP 3).
C
C              XPO        ARE THE INITIAL X, Y, Z COORDINATES OF THE
C              YPO        PARTICLE.
C              ZPO
C
C              VXPO       ARE THE INITIAL X, Y, Z VELOCITY COMPONENTS OF
C              VYPO       THE PARTICLE.
C              VZPO
C
C              XOTF       ARE THE INITIAL X, Y COORDINATES OF THE ORIGIN
C              YOTF       OF THE TRANSLATING FRAME WHICH THE TORNADO IS
C                         STATIONARY IN.
C
C              VXTF       ARE THE CONSTANT X, Y VELOCITY COMPONENTS OF THE
C              VYTF       TRANSLATING FRAME.
C
C              TO         IS THE INITIAL TIME.
C
C              TIMEIT     IS THE INTERVAL OF INTEGRATION.
C
C              HI         IS THE INITIAL TIME STEP. A GOOD VALUE TO USE
C                         IS 1.0E-4
C
C              FXTMIT     CONSTRAINS THE ODE SOLVER TO PRODUCE RESULTS FOR
C                         PRINTING AT EQUALLY SPACED TIME INTERVALS OF
C                         THIS SIZE.
C
C              EPSI       IS THE LOCAL ERROR TOLERANCE USED BY THE ODE
C                         SOLVER. A GOOD VALUE TO USE IS 1.0E-4.
C
C              THE NEXT 6 VARIABLES ARE FLAGS FOR PRODUCING PLOTS. IF
C              A FLAG IS SET TO 1 THE CORRESPONDING PLOT IS PRODUCED.
C              OTHERWISE, THE PLOT IS NOT PRODUCED.
C
C              WTPTZY     IS THE FLAG FOR A PLOT OF Z VERSES Y.
C
C              WTPTYX     IS THE FLAG FOR A PLOT OF Y VERSES X.
C
C              WTPTZX     IS THE FLAG FOR A PLOT OF Z VERSES X.
C
```

```
      C                WTPTXT    IS THE FLAG FOR A PLOT OF X VERSES TIME.
      C
      C                WTPTSX    IS THE FLAG FOR A PLOT OF SPEED VERSES X.
      C
      C                WTPTHT    IS THE FLAG FOR A PLOT OF HORIZONTAL SPEED
      C                          VERSES TIME.
      C
      C                WTVALD    IS A FLAG WHICH IF SET TO 1 CAUSES VALIDATION
      C                          INFORMATION TO BE PRINTED IN ADDITION TO THE
      C                          USUAL OUTPUT AT EACH PRINT STEP. OTHERWISE,
      C                          NO EXTRA OUTPUT IS GENERATED.
      C
      C      DECK STRUCTURE.
      C
      C         COMMENT  WHAT FOLLOWS EACH INDEX NUMBER BELOW MUST START A NEW
      C                  CARD.
      C
      C         INDEX.          CARDS.                                  FORMAT.
      C
      C         1        GROUP 1.                                       SEE ABOVE.
      C
      C         2        GROUP 2.                                       SEE ABOVE.
      C
      C         3        GROUP 3.                                       SEE ABOVE.
      C
      C         4        SENTINEL.                                      I2
      C
      C                     STLOOP (FIRST USE).
      C
      C                        STLOOP   IS AN INTEGER VARIABLE SET TO A
      C                                 SENTINEL VALUE. IF STLOOP IS 1 THEN THE
      C                                 CARDS THAT FOLLOW ARE OBTAINED BY
      C                                 RECURSIVELY STARTING AT INDEX 3.
      C                                 OTHERWISE, THE NEXT SENTINEL CARD IS
      C                                 READ.
      C
      C        5         SENTINEL.                                      I2
      C
      C                     STLOOP (SECOND USE).
      C
      C                        STLOOP   IS AN INTEGER VARIABLE SET TO A
      C                                 SENTINEL VALUE. IF STLOOP IS 1 THEN THE
      C                                 CARDS THAT FOLLOW ARE OBTAINED BY
      C                                 RECURSIVELY STARTING AT INDEX 2.
      C                                 OTHERWISE, THE NEXT SENTINEL CARD IS
      C                                 READ.
      C
      C         6        SENTINEL.                                      I2
      C
      C                     STLOOP (THIRD USE).
      C
      C                        STLOOP   IS AN INTEGER VARIABLE SET TO A
      C                                 SENTINEL VALUE. IF STLOOP IS 1 THEN THE
      C                                 CARDS THAT FOLLOW ARE OBTAINED BY
      C                                 RECUSRIVELY STARTING AT INDEX 1.
      C                                 OTHERWISE, THE RUN IS COMPLETED.
      C
      C      EXAMPLE.
```

```
C
C        GROUP 1 CARD(S).
C        GROUP 2 CARD.
C        GROUP 3 CARDS.
C         1
C        GROUP 3 CARDS.
C         0
C         1
C        GROUP 2 CARD.
C        GROUP 3 CARDS.
C         0
C         0
C         1
C        GROUP 1 CARD(S).
C        GROUP 2 CARD.
C        GROUP 3 CARDS.
C        .
C        .
C        .
C         0
C         0
C         0
C
C     STRUCTURE OF USER SUPPLIED SUBPROGRAMS.
C
C        THE DEFINITION OF THE CALLING PARAMETERS USED IN USER SUPPLIED
C        SUBROUTINES TORWDF AND DRAG CAN BE OBTAINED FROM THE CURRENTLY
C        SUPPLIED VERSIONS OF THESE ROUTINES.
C
C-----------------------------------------------------------------------
C
C           SUBROUTINE TORWDF(RPTF, COSPTF, SINPTF, ZPTF, VRWTF, VAWTF,
C          *                  VZWTF, WHICHG)
C
C              DECLARATIONS.
C
C           GO TO (100, 500, 1000), WHICHG
C
C       100    SECTION FOR READING IN PARAMETERS TO BE USED IN THIS
C              SUBROUTINE.
C
C           RETURN
C
C       500    SECTION FOR COMPUTING THE VELOCITY COMPONENTS OF THE
C              TORNADO WIND FIELD.
C
C           RETURN
C
C      1000    SECTION FOR PRINTING ANY RELEVANT PARAMETERS USED IN THE
C              COMPUTATION.
C
C           RETURN
C           END
C
C-----------------------------------------------------------------------
C
C           SUBROUTINE DRAG(PARMTS, DRAGCF, WHICHG)
C
```

```
C                    DECLARATIONS.
C
C               GO TO (100, 500, 1000), WHICHG
C
C         100        SECTION FOR READING IN PARAMETERS TO BE USED IN THIS
C                    SUBROUTINE.
C
C               RETURN
C
C         500        SECTION FOR COMPUTING THE DRAG COEFFICIENT.
C
C               RETURN
C
C        1000        SECTION FOR PRINTING ANY RELEVANT PARAMETERS USED IN THE
C                    COMPUTATION.
C
C               RETURN
C               END
C
C-----------------------------------------------------------------------
C
C  PROGRAM USAGE (OUTPUT).
C
C     LAYOUT OF OUTPUT.
C
C          THE PROGRAM OUTPUT IS BROKEN INTO 4 SECTIONS.
C
C          SECTION 1.  PROBLEM DESCRIPTION.
C
C               THE FOLLOWING DESCRIPTIVE INFORMATION IS PRINTED.
C
C                    WIND VELOCITY PARAMETERS.
C
C                         ALL THE PARAMETERS WHICH ARE INPUTED FOR COMPUTED THE
C                         TORNADO WIND FIELD ARE PRINTED. THE NUMBER, NAMES, AND
C                         MEANING OF THE PARAMETERS ARE DETERMINED BY THE USER.
C
C                    DRAG COEFFICIENT PARAMETERS.
C
C                         ALL THE PARAMETERS WHICH ARE INPUTED FOR COMPUTING THE
C                         DRAG COEFFICIENT ARE PRINTED. THE NUMBER, NAMES, AND
C                         MEANING OF THE PARAMETERS ARE DETERMINED BY THE USER.
C
C                    PARTICLE PARAMETERS.
C
C                         SELF-EXPLANATORY. FROM INPUT.
C
C                    INITIAL CONDITIONS (TORNADO WIND FIELD).
C
C                         SELF-EXPLANATORY. FROM INPUT.
C
C                    INITIAL CONDITIONS (PARTICLE).
C
C                         SELF-EXPLANATORY. FROM INPUT.
C
C          SECTION 2.  TABULAR RESULTS.
C
C               FIRST, THE INITIAL TIME AND THE TIME INTERVAL FOR OUTPUT ARE
```

```
C             PRINTED. NEXT, 13 COLUMNS OF OUTPUT ARE GENERATED. EACH LINE
C             CORRESPONDS TO A SINGLE PRINT STEP. THE MEANING OF EACH
C             COLUMN IS AS FOLLOWS.
C
C                NSTP       IS THE INTEGRATION STEP NUMBER REACHED AT TIME
C                           T.
C
C                T          IS THE SIMULATED TIME AT WHICH OUTPUT IS
C                           GENERATED.
C
C                XP         ARE THE X, Y, Z COORDINATES OF THE PARTICAL IN
C                YP         THE ABSOLUTE FRAME AT TIME T.
C                ZP
C
C                R(XY)      IS SQRT(XP ** 2 + YP ** 2).
C
C                VXP        ARE THE X, Y, Z COMPONENTS OF THE PARTICLE
C                VYP        VELOCITY IN THE ABSOLUTE FRAME AT TIME T.
C                VZP
C
C                HSPEED     IS SQRT(VXP ** 2 + VYP ** 2)
C
C                SPEED      IS SQRT(VXP ** 2 + VYP ** 2 + VZP ** 2)
C
C                HWSPEED    IS THE HORIZONTAL SPEED OF THE TORNADO WIND AT
C                           THE PARTICLE IN THE ABSOLUTE FRAME.
C
C                RTF(XY)    IS THE HORIZONTAL DISTANCE OF THE PARTICLE FROM
C                           THE ORIGIN OF THE TRANSLATING FRAME.
C
C             IF WTVALD = 1 THEN ADDITIONAL OUTPUT IS GENERATED AT EACH
C             PRINT STEP. SEE THE VALIDATION SECTION FOR THE MEANING OF
C             THIS ADDITIONAL OUTPUT.
C
C          SECTION 3.  PROBLEM TERMINATION.
C
C             FIRST, THE REASON FOR TERMINATION IS PRINTED. IT CAN BE FOR
C             ONE OF THREE REASONS.
C
C                1) THE FINAL TIME HAS BEEN REACHED.
C
C                2) THE PARTICLE HIT THE GROUND.
C
C                3) THE ODE SOLVER FAILED PREMATURELY.
C
C             NEXT, RESULTS RELEVANT TO THE WHOLE RUN ARE PRINTED. EACH IS
C             CURRENTLY TAKEN TO BE THE MAXIMUM IN ABSOLUTE VALUE OVER
C             VALUES GENERATED AT THE PRINT STEPS. THEY HAVE THE FOLLOWING
C             MEANING.
C
C                MAXVXP     ARE THE MAXIMUM X, Y, Z COMPONENTS IN ABSOLUTE
C                MAXVYP     VALUE OF THE PARTICLE VELOCITY IN THE ABSOLUTE
C                MAXVZP     FRAME.
C
C                MAXMVP     IS THE MAXIMUM SPEED OF THE PARTICLE IN THE
C                           ABSOLUTE FRAME.
C
C          SECTION 4.  PLOTS.
```

```
C
C           ZERO OR MORE PLOTS ARE PRODUCED ONE PER PAGE. THE ORDINATE
C           AND ABSCISSA LABELS CAN BE FOUND CENTERED BELOW THE GRAPH.
C           THE PLOTTING CHARACTER IS THE LETTER X. AS NOTED IN THE
C           SECTION, APPLICABILITY AND RESTRICTIONS, AT MOST 101 DATA
C           POINTS ARE PLOTTED.
C
C   ERROR HANDLING.
C
C      INPUT CHECKING.
C
C         THE FOLLOWING VARIABLES ARE CHECKED FOR VALIDITY IN THE MAIN
C         PROGRAM. IF AT LEAST ONE IS INVALID AN ERROR MESSAGE IS PRINTED
C         AND THEY ARE ALL RESET TO DEFAULT VALUES. THE DEFAULT VALUES
C         CAN BE FOUND IN THE SECTION, MACHINE/SYSTEM DEPENDENT FEATURES,
C         AT THE START OF THE EXECUTABLE CODE IN THIS MAIN PROGRAM.
C
C            HI        IS THE INITIAL STEP. IT MUST BE POSITIVE AND LESS
C                      THAN FXTMIT.
C
C            FXTMIT    IS THE TIME INTERVAL USED FOR OUTPUT. IT MUST BE
C                      POSITIVE.
C
C      PROGRAM DETECTABLE ERRORS.
C
C         AFTER EACH INTEGRATION STEP THE VARIABLE IER IS CHECKED. IF
C         ITS VALUE INDICATES THAT THE LAST STEP FAILED THEN INTEGRATION
C         TERMINATES PREMATURELY. THE REST OF THE OUTPUT IS GENERATED
C         AS USUAL WITH THE ERROR CODE PRINTED FOR THE REASON. THE OUTPUT
C         WILL REFLECT ONLY RESULTS GENERATED UP TO THE TIME OF THE
C         ERROR.
C
C   DISCUSSION OF METHOD AND ALGORITHM.
C
C      THE PROBLEM TO BE SOLVED IS A NUMERICAL INITIAL VALUE PROBLEM IN
C      ORDINARY DIFFERENTIAL EQUATIONS. SIX, FIRST ORDER, ORDINARY
C      DIFFERENTIAL EQUATIONS, REPRESENTING THE EQUATIONS OF MOTION, ARE
C      INTEGRATED FROM SOME INITIAL TIME WITH SPECIFIED INITIAL
C      CONDITIONS TO SOME LATER TIME WHICH SATISFIES SOME TERMINATION
C      CONDITION.
C
C      THE ODE SOLVER, VOADAM, IS BASED ON THE VARIABLE ORDER, VARIABLE
C      STEP ADAMS CODE DESIGNED AND IMPLEMENTATED BY C. W. GEAR IN
C      REFERENCE 1). SINCE THE SYSTEM OF ODES TO BE SOLVED IS NON-STIFF
C      ALL PARAMETERS AND CALLS TO SUBROUTINES REQUIRED FOR SOLVING
C      STIFF SYSTEMS HAVE BEEN DELETED.
C
C      ON EACH CALL TO VOADAM THE ODE SOLVER IS ASKED TO INTEGRATE THE
C      SYSTEM OF ODES OVER A STEP OF LENGTH H. THE VALUE OF H ARISES FROM
C      ONE OF TWO SOURCES.
C
C         1) THE VALUE OF H RETURNED BY THE PREVIOUS CALL OF VOADAM.
C
C         2) THE VALUE OF H SPECIFIED BY CALLER INTERACTION.
C
C      SOURCE 2) IS USED ONLY FOR THE INITIAL STEP AND WHEN THE STEP
C      IS MODIFIED SO THAT IT FALLS ON A MULTIPLE OF THE FIXED TIME
C      INTERVAL USED FOR PRINTING. OTHERWISE, SOURCE 1) IS USED SO AS TO
```

```
C     ACHIEVE AN ECONOMICAL INTEGRATION.
C
C     THE INPUT VALUE OF H IS USED UNLESS THE ERROR CRITERIA CANNOT BE
C     MET. IN THIS CASE THE STEP AND/OR ORDER ARE MODIFIED TO TRY TO
C     MEET THE ERROR CRITERIA. IF AN ATTEMPT IS MADE TO REDUCE THE STEP
C     BELOW A CALLER SUPPLIED VALUE, HMIN, THE ODE SOLVER QUITS AND
C     RETURNS THE APPROPRIATE NONZERO ERROR CONDITION CODE.
C
C     ONCE A SUCCESSFUL STEP HAS BEEN TAKEN, VOADAM ESTIMATES AND
C     RETURNS A GOOD VALUE OF H TO BE USED FOR THE NEXT STEP. THIS
C     ESTIMATED STEP CANNOT BE GREATER THAN A CALLER SUPPLIED VALUE,
C     HMAX.
C
C     FOR MORE DETAILS ON VOADAM CONSULT THE MACHINE READABLE
C     DOCUMENTATION AT THE BEGINNING OF THE SUBROUTINE.
C
C  VALIDATION.
C
C     THE PROGRAM PROVIDES THE USER BY WAY OF THE WTVALD INPUT VARIABLE
C     THE CAPABILITY OF PRINTING IMPORTANT INTERMEDIATE QUANTITIES.
C     THESE QUANTITIES ARE PRINTED AS ADDITIONAL OUTPUT AT THE NORMAL
C     PRINTING STEPS. THE AMOUNT OF OUTPUT PER PRINT STEP INCREASES FROM
C     ONE LINE TO SEVEN LINES. TO INTERPRET THE MEANING OF THE VARIABLES
C     PRINTED CONSULT THE TABLE BELOW ACCOMPANIED BY APPENDIX A OF
C     REFERENCE 2).
C
C        RTF       IS THE CYLINDRICAL RADIUS OF THE ORIGIN OF THE
C                  TRANSLATING FRAME FROM THE ORIGIN OF THE ABSOLUTE
C                  FRAME.
C
C        COSTF     ARE THE COSINE, SINE OF THE ANGLE THAT THE DIRECTION
C        SINTF     OF TRANSLATION MAKES WITH THE X AXIS OF THE ABSOLUTE
C                  FRAME.
C
C        PRTF      IS THE CYLINDRICAL RADIUS OF THE PARTICLE FROM THE
C                  ORIGIN OF THE TRANSLATING FRAME.
C
C        PCOSTF    ARE THE COSINE, SINE OF THE ANGLE THAT THE CYLINDRICAL
C        PSINTF    RADIUS TO THE PARTICLE MAKES WITH THE X AXIS OF THE
C                  TRANSLATING FRAME.
C
C        WRTF      ARE THE RADIAL, ANGULAR, AND Z COMPONENTS OF THE WIND
C        WANGTF    VELOCITY AT THE PARTICLE USING THE REVOLVING FRAME.
C        WZTF
C
C        WXTF      ARE THE X, Y COMPONENTS OF THE WIND VELOCITY AT THE
C        WYTF      PARTICLE USING THE TRANSLATING FRAME.
C
C        WXAF      ARE THE X, Y, Z COMPONENTS OF THE WIND VELOCITY AT THE
C        WYAF      PARTICLE USING THE ABSOLUTE FRAME.
C        WZAF
C
C        RVXPAF    ARE THE X, Y, Z COMPONENTS OF THE RELATIVE VELOCITY OF
C        RVYPAF    THE PARTICLE WITH RESPECT TO THE WIND USING THE
C        RVZPAF    ABSOLUTE FRAME.
C
C
C  PORTABILITY.
```

```
C
C     LANGUAGE.
C
C        BELL VERIFIER FORTRAN.
C
C     PRECISION.
C
C        SINGLE.
C
C     RESTRICTIONS.
C
C        THIS PROGRAM WAS DESIGNED TO RUN CORRECTLY WITH A MINIMUM OF
C        MODIFICATION ON MACHINES WHICH HAVE A SINGLE PRECISION
C        FLOATING POINT WORD WITH A MANTISSA IN THE RANGE OF 24 THROUGH
C        48 BITS.
C
C        ALL INPUT IS FROM FORTRAN FORMATTED CARD IMAGES AND IS READ
C        FROM LOGICAL UNIT 5.
C
C        ALL OUTPUT IS FORTRAN FORMAT GENERATED AND IS WRITTEN TO
C        LOGICAL UNIT 6. THE INTENDED DEVICE IS A LINE PRINTER SET TO
C        AT LEAST A 132 CHARACTERS PER LINE AND AT LEAST 60 LINES PER
C        PAGE.
C
C        FOR FURTHER DETAILS SEE THE SECTION, MACHINE/SYSTEM DEPENDENT
C        FEATURES, AT THE BEGINNING OF THE EXECUTABLE CODE IN THIS MAIN
C        PROGRAM AND ALL SUBPROGRAMS LISTED IN THE SUBPROGRAM DIRECTORY.
C
C  CODE RESPONSIBILITY.
C
C                             MARTIN CORDES
C                             APPLIED MATHEMATICS DIVISION
C                             NATIONAL BUREAU OF STANDARDS
C                             WASHINGTON, D.C. 20234
C
C                             (301) 921-2631
C
C  HISTORY.
C
C     ORIGINAL VERSION.
C
C        MAY 1975
C
C     REVISED VERSION(S).
C
C        AUG 1975
C        APR 1976
C
C  REFERENCES.
C
C        1) C. W. GEAR, NUMERICAL INITIAL VALUE PROBLEMS IN ORDINARY
C           DIFFERENTIAL EQUATIONS, PRENTICE-HALL, 1971, 253P.
C
C        2) E. SIMIU AND M. CORDES, TORNADO - BORNE MISSILE SPEEDS,
C           NBS INTERAGENCY REPORT
C
C-----------------------------------------------------------------------
C
```

```
TORNADO*WIND(1).TORWDF$FX(0)
            SUBROUTINE TORWDF(RPTF,COSPTF,SINPTF,ZPTF,VRWTF,VAWTF,VZWTF,
           *                   WHICHG)
      C
      C   THIS SUBROUTINE COMPUTES THE TORNADO WIND VELOCITY COMPONENTS
      C
      C   DEFINITION OF PARAMETERS.
      C   RPTF,COSPTF,SINPTF,ZPTF    = RADIAL COORDINATE, COSINE AND SINE OF
      C                                THE ANGLE, AND Z COORDINATE OF THE
      C                                PARTICLE WITH RESPECT TO THE CYLINDRICAL
      C                                COORDINATE SYSTEM DEFINED AS THE
      C                                TRANSLATING FRAME
      C   VRWTF,VAWTF,VZWTF          = RADIAL, ANGULAR, AND Z COMPONENTS OF THE
      C                                WIND AT THE PARTICLE AS MEASURED IN THE
      C                                TRANSLATING FRAME
      C   WHICHG                     = 1 - READ IN WIND VELOCITY PARAMETERS
      C                                2 - COMPUTE WIND VELOCITY COMPONENTS
      C                                3 - PRINT WIND VELOCITY PARAMETERS
      C
      C   PARAMETER DECLARATIONS.
      C
            REAL RPTF,COSPTF,SINPTF,ZPTF,VRWTF,VAWTF,VZWTF
            INTEGER WHICHG
      C
      C   OTHER COMMON VARIABLES.
      C
            REAL RMTVZO,MTV,K1,K2,K3,K4,K5
      C
            COMMON /TWDFDR/RMTVZO,MTV,K1,K2,K3,K4,K5
      C
      C   LOCAL VARIABLE DECLARATIONS.
      C
            REAL RMTVZ,R33Z,RATIO
            INTEGER ID,OD
      C
      C--------------------------------------------------------------------
      C
      C  MACHINE/SYSTEM DEPENDENT FEATURES.
      C
      C     DEFINITION OF I/O UNITS USED IN THIS SUBROUTINE.
      C
      C        ID        IS THE UNIT USED FOR INPUTING DATA.
      C
      C        OD        IS THE UNIT USED FOR OUTPUTING RESULTS.
      C
      C
            DATA ID, OD/5, 6/
      C
      C
      C--------------------------------------------------------------------
      C
            GO TO (100,500,1000),WHICHG
      C
      C   READ IN WIND VELOCITY PARAMETERS
      C
        100    READ(ID,250) RMTVZO,MTV,K1,K2,K3,K4,K5
        250    FORMAT(10E8.0)
            RETURN
```

```
      C
      C   COMPUTE WIND VELOCITY COMPONENTS
      C
        500     IF(ZPTF.GE.60.0) GO TO 525
                    RMTVZ=RMTVZ0+K1*ZPTF
                    GO TO 550
        525     RMTVZ=RMTVZ0+K1*60.0
        550     R33Z=(MTV/33.0)**.625*RMTVZ
      C
                IF(RPTF.GE.RMTVZ) GO TO 575
                    RATIO=RPTF/RMTVZ
                    GO TO 600
        575     RATIO=RMTVZ/RPTF
                IF(RPTF.GE.R33Z) GO TO 650
      C
        600     IF(ZPTF.GE.60.0) GO TO 625
                    VRWTF=-K5*(R33Z-RPTF)/(R33Z-RMTVZ)*RPTF
                    VAWTF=RATIO*MTV
                    VZWTF=K3*(R33Z-RPTF)/(R33Z-RMTVZ)*ZPTF+K4/3.0*VAWTF
                RETURN
        625     IF(ZPTF.GE.240.0) GO TO 650
                    VRWTF=-K5*(R33Z-RPTF)/(R33Z-RMTVZ)*RPTF*(240.0-ZPTF)/180.0
                    VAWTF=RATIO*MTV
                    VZWTF=(1.33-ZPTF/180.0)*(K3*(R33Z-RPTF)/(R33Z-RMTVZ)*60.0+
           *                K4/3.0*VAWTF)
                RETURN
        650         VRWTF=0.0
                    VAWTF=RATIO*MTV
                    VZWTF=0.0
                RETURN
      C
      C   PRINT WIND VELOCITY PARAMETERS
      C
       1000     WRITE(OD,1100) RMTVZ0,MTV,K1,K2,K3,K4,K5
       1100     FORMAT(26H0WIND VELOCITY PARAMETERS./
           *        5X,15H RMTVZ0        = ,1PE12.4/
           *        5X,15H MTV           = ,1PE12.4/
           *        5X,15H K1            = ,1PE12.4/
           *        5X,15H K2            = ,1PE12.4/
           *        5X,15H K3            = ,1PE12.4/
           *        5X,15H K4            = ,1PE12.4/
           *        5X,15H K5            = ,1PE12.4)
            RETURN
      C
            END
END PRT

@PRT,S F1.DRAG$EX
```

```
TORNADO*WIND(1).DRAG$EX(0)
            SUBROUTINE DRAG(PARMTS,DRAGCF,WHICHG)
      C
      C   THIS SUBROUTINE COMPUTES THE DRAG COEFFICIENT FOR THE PARTICLE
      C
      C   DEFINITION OF PARAMETERS.
      C   PARMTS(1)                     = ARRAY OF TIME DEPENDENT PARAMETERS THAT
      C                                   AFFECT THE COMPUTATION OF THE DRAG
      C                                   COEFFICIENT
      C   DRAGCF                        = DRAG COEFFICIENT COMPUTED BY THIS
      C                                   SUBROUTINE
      C   WHICH                         = 1 - READ IN DRAG COEFFICIENT PARAMETERS
      C                                   2 - COMPUTE DRAG COEFFICIENT
      C                                   3 - PRINT DRAG COEFFICIENT PARAMETERS
      C
      C   PARAMETER DECLARATIONS.
      C
            REAL PARMTS(1)
            REAL DRAGCF
            INTEGER WHICHG
      C
      C   OTHER COMMON VARIABLES.
      C
            REAL CDRAG
      C
            COMMON /CGCOPR/CDRAG
      C
      C   LOCAL VARIABLE DECLARATIONS.
      C
            INTEGER ID,OD
      C
      C-----------------------------------------------------------------------
      C
      C  MACHINE/SYSTEM DEPENDENT FEATURES.
      C
      C     DEFINITION OF I/O UNITS USED IN THIS SUBROUTINE.
      C
      C        ID        IS THE UNIT USED FOR INPUTING DATA.
      C
      C        OD        IS THE UNIT USED FOR OUTPUTING RESULTS.
      C
      C
            DATA ID, OD/5, 6/
      C
      C
      C-----------------------------------------------------------------------
      C
            GO TO (100,500,1000),WHICHG
      C
      C   READ IN DRAG COEFFICIENT PARAMETERS
      C
        100   READ(ID,250) CDRAG
        250   FORMAT(6E12.0)
            RETURN
      C
      C   COMPUTE DRAG COEFFICIENT
      C
        500   DRAGCF=CDRAG
```

```
              RETURN
       C
       C   PRINT DRAG COEFFICIENT PARAMETERS
       C
        1000    WRITE(OC,1100) CDRAG
        1100    FORMAT(29HODRAG COEFFICIENT PARAMETERS./
     *        5X,15H CDRAG        = ,1PE12.4)
              RETURN
       C
              END
  END PRT

@PFT,S F1.DATA$EX
```

```
TORNADO*WIND(1).DATA$EX(1)
     1            46.0   130.0     0.0      0.0      1.0      1.0      1.0
     2                 1.0
     3                 1.0         10.0
     4                46.0          0.0        40.0            0.0          0.0          0.0
     5                 0.0          0.0        35.0            0.0          0.0
     6                10.0
     7              1.0E-4      1.0E-1      1.0E-4
     8         1 1 1 1 1 1 0
     9         0
    10         0
    11         0
  END PRT

@XQT F1.ABS$EX
```

PROBLEM DESCRIPTION

```
WIND VELCCITY PARAMETERS.
     RMTVZO       =     4.6000+01
     MTV          =     1.3000+02
     K1           =     0.0000
     K2           =     0.0000
     K3           =     1.0000+00
     K4           =     1.CCC0+00
     K5           =     1.0000+00

DRAG COEFFICIENT PARAMETERS.
     CDRAG        =     1.CCC0+00

PARTICLE PARAMETERS.
     AREA         =     1.0000+00
     MASS         =     1.0000+01

INITIAL CONDITIONS. (TORNADO WIND FIELD)
     PCSITICN.   X =     0.0000        Y =    0.0000
     VELOCITY.  VX =     3.5000+01    VY =    0.0000

INITIAL CONDITIONS. (PARTICLE)
     PCSITION.   X =     4.6000+01     Y =    0.0000         Z =    4.0000+01
     VELOCITY.  VX =     C.0000       VY =    0.0000        VZ =    0.0000
```

TABULAR RESULTS

INITIAL TIME.   C.0000
TIME INTERVAL FOR OUTPUT.   1.0000-01

NSTP	T	XP	YP	ZP	R(XY)	VXP	VYP	VZP	HSPEED	SPEED	HWSPEED	RTF(XY)
0	0.00	4.60+01	0.00	4.00+01	4.60+01	0.00	0.00	0.00	0.00	0.00	1.30+02	4.60+01
21	1.00-01	4.56+01	3.73+00	4.24+01	4.58+01	-6.87+00	5.94+01	3.95+01	5.98+01	7.17+01	1.17+02	4.23+01
34	2.00-01	4.46+01	1.05+01	4.71+01	4.58+01	-1.33+01	7.33+01	5.31+01	7.45+01	9.15+01	1.01+02	3.91+01
48	3.00-01	4.29+01	1.80+01	5.29+01	4.65+01	-2.13+01	7.56+01	6.20+01	7.86+01	1.00+02	8.86+01	3.71+01
55	4.00-01	4.03+01	2.54+01	5.96+01	4.76+01	-3.19+01	7.07+01	7.12+01	7.75+01	1.05+02	8.08+01	3.66+01
66	5.00-01	3.65+01	3.19+01	6.71+01	4.85+01	-4.36+01	5.92+01	7.90+01	7.35+01	1.08+02	7.84+01	3.72+01
72	6.00-01	3.16+01	3.71+01	7.52+01	4.87+01	-5.42+01	4.28+01	8.35+01	6.90+01	1.08+02	8.11+01	3.86+01
80	7.00-01	2.57+01	4.04+01	8.37+01	4.79+01	-6.25+01	2.26+01	8.52+01	6.64+01	1.08+02	8.75+01	4.04+01
85	8.00-01	1.92+01	4.15+01	9.22+01	4.58+01	-6.73+01	7.09-02	8.49+01	6.73+01	1.08+02	9.64+01	4.24+01
90	9.00-01	1.24+01	4.04+01	1.01+02	4.22+01	-6.80+01	-2.38+01	8.32+01	7.20+01	1.10+02	1.07+02	4.46+01
102	1.00+00	5.79+00	3.68+01	1.09+02	3.72+01	-6.37+01	-4.76+01	8.03+01	7.96+01	1.13+02	1.14+02	4.70+01
110	1.10+00	-8.55-02	3.10+01	1.17+02	3.10+01	-5.27+01	-6.76+01	7.61+01	8.57+01	1.15+02	1.15+02	4.95+01
118	1.20+00	-4.59+00	2.34+01	1.24+02	2.39+01	-3.67+01	-8.27+01	7.10+01	9.04+01	1.15+02	1.17+02	5.21+01
122	1.30+00	-7.36+00	1.46+01	1.31+02	1.64+01	-1.85+01	-9.24+01	6.56+01	9.43+01	1.15+02	1.18+02	5.48+01
127	1.40+00	-8.28+00	5.08+00	1.37+02	9.72+00	-1.15-01	-9.74+01	6.01+01	9.74+01	1.14+02	1.18+02	5.75+01
133	1.50+00	-7.42+00	-4.75+00	1.43+02	8.81+00	1.72+01	-9.86+01	5.50+01	1.00+02	1.14+02	1.19+02	6.01+01
137	1.60+00	-4.90+00	-1.45+01	1.48+02	1.53+01	3.28+01	-9.68+01	5.02+01	1.02+02	1.14+02	1.19+02	6.26+01
142	1.70+00	-9.26-01	-2.40+01	1.53+02	2.41+01	4.64+01	-9.30+01	4.58+01	1.04+02	1.14+02	1.18+02	6.50+01
146	1.80+00	4.31+00	-3.31+01	1.57+02	3.34+01	5.80+01	-8.78+01	4.17+01	1.05+02	1.13+02	1.18+02	6.74+01
151	1.90+00	1.06+01	-4.16+01	1.61+02	4.29+01	6.78+01	-8.17+01	3.81+01	1.06+02	1.13+02	1.17+02	6.97+01
153	2.00+00	1.78+01	-4.94+01	1.65+02	5.25+01	7.59+01	-7.51+01	3.47+01	1.07+02	1.12+02	1.16+02	7.19+01
157	2.10+00	2.58+01	-5.66+01	1.68+02	6.22+01	8.25+01	-6.83+01	3.17+01	1.07+02	1.12+02	1.14+02	7.40+01
162	2.20+00	3.43+01	-6.31+01	1.71+02	7.18+01	8.79+01	-6.14+01	2.89+01	1.07+02	1.11+02	1.13+02	7.62+01
164	2.30+00	4.33+01	-6.89+01	1.74+02	8.13+01	9.21+01	-5.46+01	2.64+01	1.07+02	1.10+02	1.11+02	7.83+01
168	2.40+00	5.27+01	-7.40+01	1.77+02	9.08+01	9.53+01	-4.80+01	2.40+01	1.07+02	1.09+02	1.10+02	8.04+01

NSTP	T	XP	YP	ZP	R(XY)	VXP	VYP	VZP	HSPEED	SPEED	HWSPEED	RTF(XY)
170	2.50+00	6.23+01	-7.85+01	1.79+02	1.00+02	9.77+01	-4.16+01	2.19+01	1.06+02	1.08+02	1.08+02	8.24+01
174	2.60+00	7.22+01	-8.23+01	1.81+02	1.09+02	9.94+01	-3.55+01	1.99+01	1.06+02	1.07+02	1.07+02	8.44+01
178	2.70+00	8.22+01	-8.56+01	1.83+02	1.19+02	1.00+02	-2.98+01	1.80+01	1.05+02	1.06+02	1.05+02	8.65+01
180	2.80+00	9.23+01	-8.83+01	1.85+02	1.28+02	1.01+02	-2.44+01	1.63+01	1.04+02	1.05+02	1.03+02	8.85+01
183	2.90+00	1.02+02	-9.05+01	1.86+02	1.37+02	1.01+02	-1.94+01	1.48+01	1.03+02	1.04+02	1.01+02	9.05+01
187	3.00+00	1.12+02	-9.22+01	1.87+02	1.45+02	1.01+02	-1.47+01	1.33+01	1.02+02	1.03+02	9.96+01	9.25+01
190	3.10+00	1.23+02	-9.34+01	1.89+02	1.54+02	1.01+02	-1.04+01	1.19+01	1.01+02	1.02+02	9.79+01	9.45+01
194	3.20+00	1.33+02	-9.43+01	1.90+02	1.63+02	9.98+01	-6.33+00	1.06+01	1.00+02	1.01+02	9.62+01	9.65+01
199	3.30+00	1.43+02	-9.47+01	1.91+02	1.71+02	9.89+01	-2.62+00	9.37+00	9.89+01	9.93+01	9.45+01	9.85+01
201	3.40+00	1.52+02	-9.48+01	1.92+02	1.79+02	9.78+01	7.82-01	8.22+00	9.78+01	9.81+01	9.29+01	1.01+02
204	3.50+00	1.62+02	-9.46+01	1.93+02	1.88+02	9.66+01	3.90+00	7.13+00	9.67+01	9.69+01	9.13+01	1.03+02
208	3.60+00	1.72+02	-9.40+01	1.93+02	1.96+02	9.53+01	6.74+00	6.09+00	9.55+01	9.57+01	8.97+01	1.05+02
210	3.70+00	1.81+02	-9.32+01	1.94+02	2.04+02	9.40+01	9.32+00	5.10+00	9.44+01	9.46+01	8.82+01	1.07+02
213	3.80+00	1.90+02	-9.22+01	1.94+02	2.12+02	9.26+01	1.17+01	4.07+00	9.33+01	9.34+01	8.68+01	1.09+02
218	3.90+00	2.00+02	-9.09+01	1.95+02	2.19+02	9.11+01	1.39+01	2.64+00	9.22+01	9.22+01	8.51+01	1.11+02
222	4.00+00	2.09+02	-8.94+01	1.95+02	2.27+02	8.95+01	1.61+01	1.40+00	9.10+01	9.10+01	8.36+01	1.13+02
224	4.10+00	2.18+02	-8.77+01	1.95+02	2.35+02	8.79+01	1.81+01	3.14-01	8.98+01	8.98+01	8.20+01	1.15+02
228	4.20+00	2.26+02	-8.58+01	1.95+02	2.42+02	8.62+01	2.00+01	-6.44-01	8.85+01	8.85+01	8.05+01	1.17+02
232	4.30+00	2.35+02	-8.37+01	1.95+02	2.49+02	8.45+01	2.18+01	-1.49+00	8.73+01	8.73+01	7.90+01	1.19+02
234	4.40+00	2.43+02	-8.14+01	1.95+02	2.56+02	8.28+01	2.34+01	-2.24+00	8.60+01	8.60+01	7.76+01	1.21+02
237	4.50+00	2.51+02	-7.90+01	1.94+02	2.63+02	8.10+01	2.50+01	-2.91+00	8.48+01	8.48+01	7.62+01	1.23+02
242	4.60+00	2.59+02	-7.64+01	1.94+02	2.70+02	7.92+01	2.64+01	-3.50+00	8.35+01	8.36+01	7.48+01	1.25+02
245	4.70+00	2.67+02	-7.37+01	1.94+02	2.77+02	7.75+01	2.77+01	-4.04+00	8.23+01	8.24+01	7.34+01	1.26+02
250	4.80+00	2.75+02	-7.09+01	1.93+02	2.84+02	7.57+01	2.89+01	-4.52+00	8.11+01	8.12+01	7.21+01	1.28+02
253	4.90+00	2.82+02	-6.80+01	1.93+02	2.90+02	7.40+01	3.00+01	-4.96+00	7.99+01	8.00+01	7.09+01	1.30+02
257	5.00+00	2.90+02	-6.49+01	1.92+02	2.97+02	7.23+01	3.10+01	-5.35+00	7.87+01	7.89+01	6.96+01	1.32+02
262	5.10+00	2.97+02	-6.18+01	1.92+02	3.03+02	7.07+01	3.19+01	-5.71+00	7.75+01	7.77+01	6.84+01	1.33+02
264	5.20+00	3.04+02	-5.85+01	1.91+02	3.09+02	6.90+01	3.28+01	-6.05+00	7.64+01	7.66+01	6.73+01	1.35+02
268	5.30+00	3.11+02	-5.52+01	1.90+02	3.15+02	6.74+01	3.35+01	-6.35+00	7.53+01	7.55+01	6.61+01	1.37+02
270	5.40+00	3.17+02	-5.18+01	1.90+02	3.21+02	6.58+01	3.42+01	-6.63+00	7.42+01	7.45+01	6.50+01	1.38+02
274	5.50+00	3.24+02	-4.84+01	1.89+02	3.27+02	6.43+01	3.48+01	-6.89+00	7.31+01	7.34+01	6.39+01	1.40+02
276	5.60+00	3.30+02	-4.49+01	1.88+02	3.33+02	6.28+01	3.53+01	-7.14+00	7.20+01	7.24+01	6.29+01	1.41+02
279	5.70+00	3.36+02	-4.13+01	1.88+02	3.39+02	6.13+01	3.58+01	-7.36+00	7.10+01	7.13+01	6.18+01	1.43+02
283	5.80+00	3.42+02	-3.77+01	1.87+02	3.44+02	5.98+01	3.62+01	-7.58+00	6.99+01	7.03+01	6.08+01	1.44+02
285	5.90+00	3.48+02	-3.41+01	1.86+02	3.50+02	5.84+01	3.65+01	-7.78+00	6.89+01	6.93+01	5.98+01	1.46+02
289	6.00+00	3.54+02	-3.04+01	1.85+02	3.55+02	5.70+01	3.69+01	-7.96+00	6.79+01	6.84+01	5.88+01	1.47+02
292	6.10+00	3.60+02	-2.67+01	1.85+02	3.61+02	5.57+01	3.71+01	-8.14+00	6.69+01	6.74+01	5.79+01	1.49+02
296	6.20+00	3.65+02	-2.30+01	1.84+02	3.66+02	5.44+01	3.74+01	-8.31+00	6.59+01	6.65+01	5.69+01	1.50+02
300	6.30+00	3.71+02	-1.92+01	1.83+02	3.71+02	5.31+01	3.75+01	-8.46+00	6.50+01	6.55+01	5.60+01	1.51+02
304	6.40+00	3.76+02	-1.55+01	1.82+02	3.76+02	5.18+01	3.77+01	-8.61+00	6.41+01	6.46+01	5.51+01	1.53+02
309	6.50+00	3.81+02	-1.17+01	1.81+02	3.81+02	5.06+01	3.78+01	-8.76+00	6.31+01	6.37+01	5.42+01	1.54+02
312	6.60+00	3.86+02	-7.92+00	1.80+02	3.86+02	4.94+01	3.79+01	-8.89+00	6.22+01	6.29+01	5.34+01	1.55+02
316	6.70+00	3.91+02	-4.13+00	1.79+02	3.91+02	4.82+01	3.79+01	-9.02+00	6.13+01	6.20+01	5.25+01	1.56+02
319	6.80+00	3.96+02	-3.38-01	1.78+02	3.96+02	4.71+01	3.79+01	-9.14+00	6.04+01	6.11+01	5.17+01	1.58+02
323	6.90+00	4.00+02	3.45+00	1.78+02	4.00+02	4.59+01	3.79+01	-9.26+00	5.96+01	6.03+01	5.09+01	1.59+02
327	7.00+00	4.05+02	7.24+00	1.77+02	4.05+02	4.49+01	3.79+01	-9.37+00	5.87+01	5.95+01	5.00+01	1.60+02
330	7.10+00	4.09+02	1.10+01	1.76+02	4.09+02	4.38+01	3.78+01	-9.48+00	5.79+01	5.86+01	4.92+01	1.61+02
334	7.20+00	4.14+02	1.48+01	1.75+02	4.14+02	4.28+01	3.77+01	-9.58+00	5.70+01	5.78+01	4.85+01	1.62+02
339	7.30+00	4.18+02	1.86+01	1.74+02	4.18+02	4.18+01	3.76+01	-9.68+00	5.62+01	5.70+01	4.77+01	1.63+02
343	7.40+00	4.22+02	2.23+01	1.73+02	4.22+02	4.08+01	3.75+01	-9.77+00	5.54+01	5.62+01	4.69+01	1.64+02

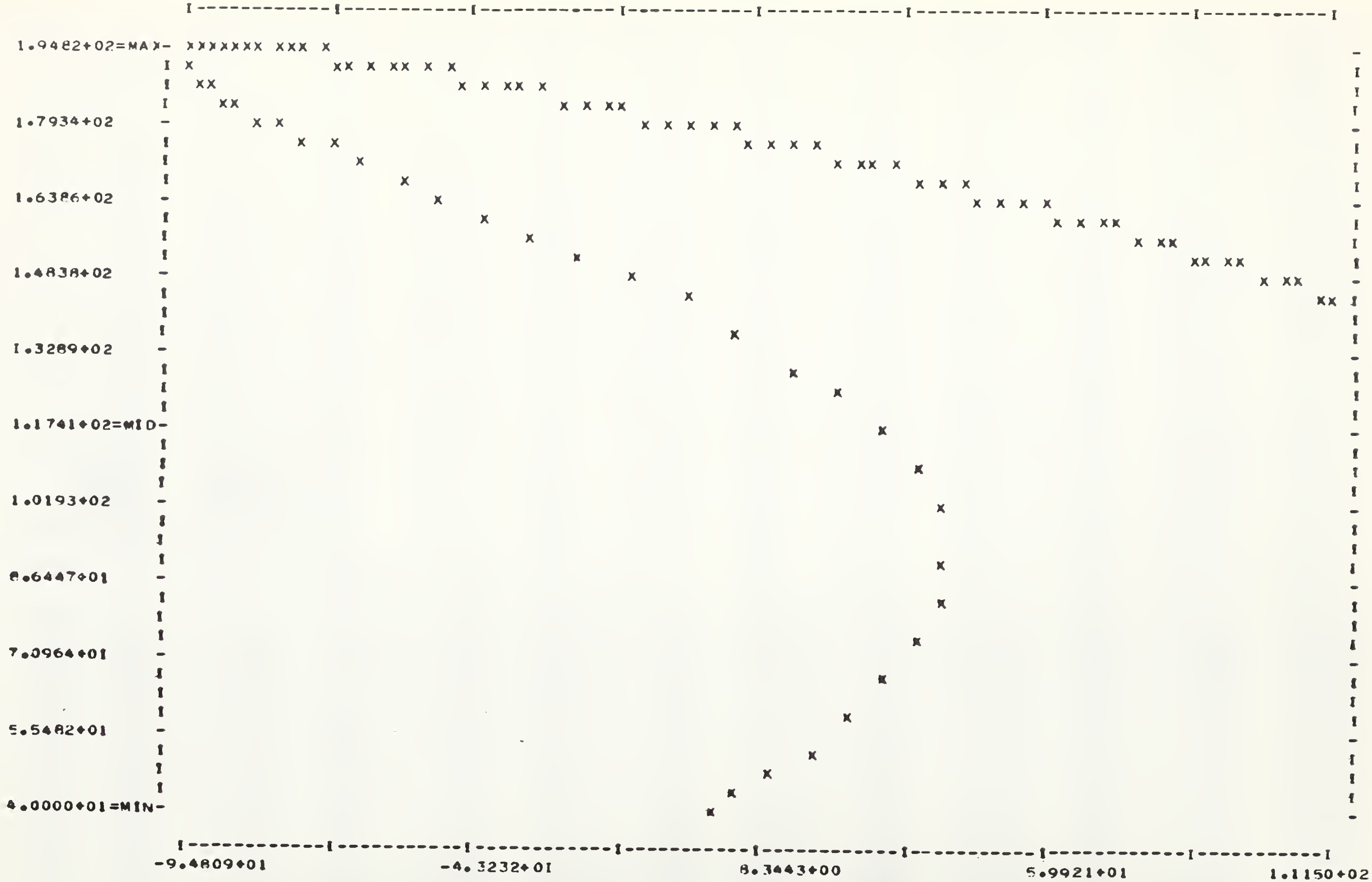
NSTP	T	XP	YP	ZP	R(XY)	VXP	VYP	VZP	HSPEED	SPEED	HWSPEED	RTF(XY)
345	7.50+00	4.26+02	2.61+01	1.72+02	4.27+02	3.98+01	3.74+01	-9.86+00	5.46+01	5.55+01	4.62+01	1.65+02
348	7.60+00	4.30+02	2.98+01	1.71+02	4.31+02	3.89+01	3.72+01	-9.94+00	5.38+01	5.47+01	4.54+01	1.67+02
352	7.70+00	4.34+02	3.35+01	1.70+02	4.35+02	3.79+01	3.70+01	-1.00+01	5.30+01	5.40+01	4.47+01	1.68+02
356	7.80+00	4.37+02	3.72+01	1.69+02	4.39+02	3.71+01	3.68+01	-1.01+01	5.22+01	5.32+01	4.40+01	1.69+02
360	7.90+00	4.41+02	4.09+01	1.68+02	4.43+02	3.62+01	3.66+01	-1.02+01	5.15+01	5.25+01	4.33+01	1.70+02
364	8.00+00	4.45+02	4.45+01	1.67+02	4.47+02	3.53+01	3.64+01	-1.03+01	5.07+01	5.18+01	4.26+01	1.71+02
366	8.10+00	4.48+02	4.82+01	1.66+02	4.51+02	3.45+01	3.62+01	-1.03+01	5.00+01	5.10+01	4.19+01	1.72+02
370	8.20+00	4.52+02	5.18+01	1.65+02	4.55+02	3.37+01	3.59+01	-1.04+01	4.92+01	5.03+01	4.12+01	1.73+02
374	8.30+00	4.55+02	5.53+01	1.64+02	4.58+02	3.29+01	3.57+01	-1.05+01	4.85+01	4.96+01	4.05+01	1.73+02
376	8.40+00	4.58+02	5.89+01	1.63+02	4.62+02	3.21+01	3.54+01	-1.05+01	4.78+01	4.89+01	3.99+01	1.74+02
380	8.50+00	4.61+02	6.24+01	1.62+02	4.66+02	3.14+01	3.51+01	-1.06+01	4.71+01	4.83+01	3.92+01	1.75+02
383	8.60+00	4.64+02	6.59+01	1.60+02	4.69+02	3.06+01	3.48+01	-1.06+01	4.64+01	4.76+01	3.86+01	1.76+02
387	8.70+00	4.67+02	6.94+01	1.59+02	4.73+02	2.99+01	3.45+01	-1.07+01	4.57+01	4.69+01	3.79+01	1.77+02
391	8.80+00	4.70+02	7.28+01	1.58+02	4.76+02	2.92+01	3.42+01	-1.07+01	4.50+01	4.63+01	3.73+01	1.78+02
395	8.90+00	4.73+02	7.62+01	1.57+02	4.79+02	2.85+01	3.39+01	-1.08+01	4.43+01	4.56+01	3.67+01	1.79+02
397	9.00+00	4.76+02	7.96+01	1.56+02	4.83+02	2.79+01	3.36+01	-1.08+01	4.37+01	4.50+01	3.61+01	1.80+02
401	9.10+00	4.79+02	8.30+01	1.55+02	4.86+02	2.72+01	3.33+01	-1.09+01	4.30+01	4.43+01	3.54+01	1.81+02
404	9.20+00	4.82+02	8.63+01	1.54+02	4.89+02	2.66+01	3.29+01	-1.09+01	4.23+01	4.37+01	3.48+01	1.81+02
408	9.30+00	4.84+02	8.95+01	1.53+02	4.92+02	2.60+01	3.26+01	-1.10+01	4.17+01	4.31+01	3.42+01	1.82+02
411	9.40+00	4.87+02	9.28+01	1.52+02	4.96+02	2.54+01	3.23+01	-1.10+01	4.10+01	4.25+01	3.37+01	1.83+02
415	9.50+00	4.89+02	9.60+01	1.51+02	4.99+02	2.48+01	3.19+01	-1.11+01	4.04+01	4.19+01	3.31+01	1.84+02
418	9.60+00	4.92+02	9.92+01	1.50+02	5.02+02	2.42+01	3.15+01	-1.11+01	3.98+01	4.13+01	3.25+01	1.85+02
422	9.70+00	4.94+02	1.02+02	1.48+02	5.05+02	2.37+01	3.12+01	-1.11+01	3.91+01	4.07+01	3.19+01	1.85+02
424	9.80+00	4.96+02	1.05+02	1.47+02	5.08+02	2.31+01	3.08+01	-1.12+01	3.85+01	4.01+01	3.14+01	1.86+02
427	9.90+00	4.99+02	1.08+02	1.46+02	5.10+02	2.26+01	3.04+01	-1.12+01	3.79+01	3.95+01	3.08+01	1.87+02
429	1.00+01	5.01+02	1.11+02	1.45+02	5.13+02	2.21+01	3.01+01	-1.12+01	3.73+01	3.90+01	3.03+01	1.88+02

PROBLEM TERMINATION

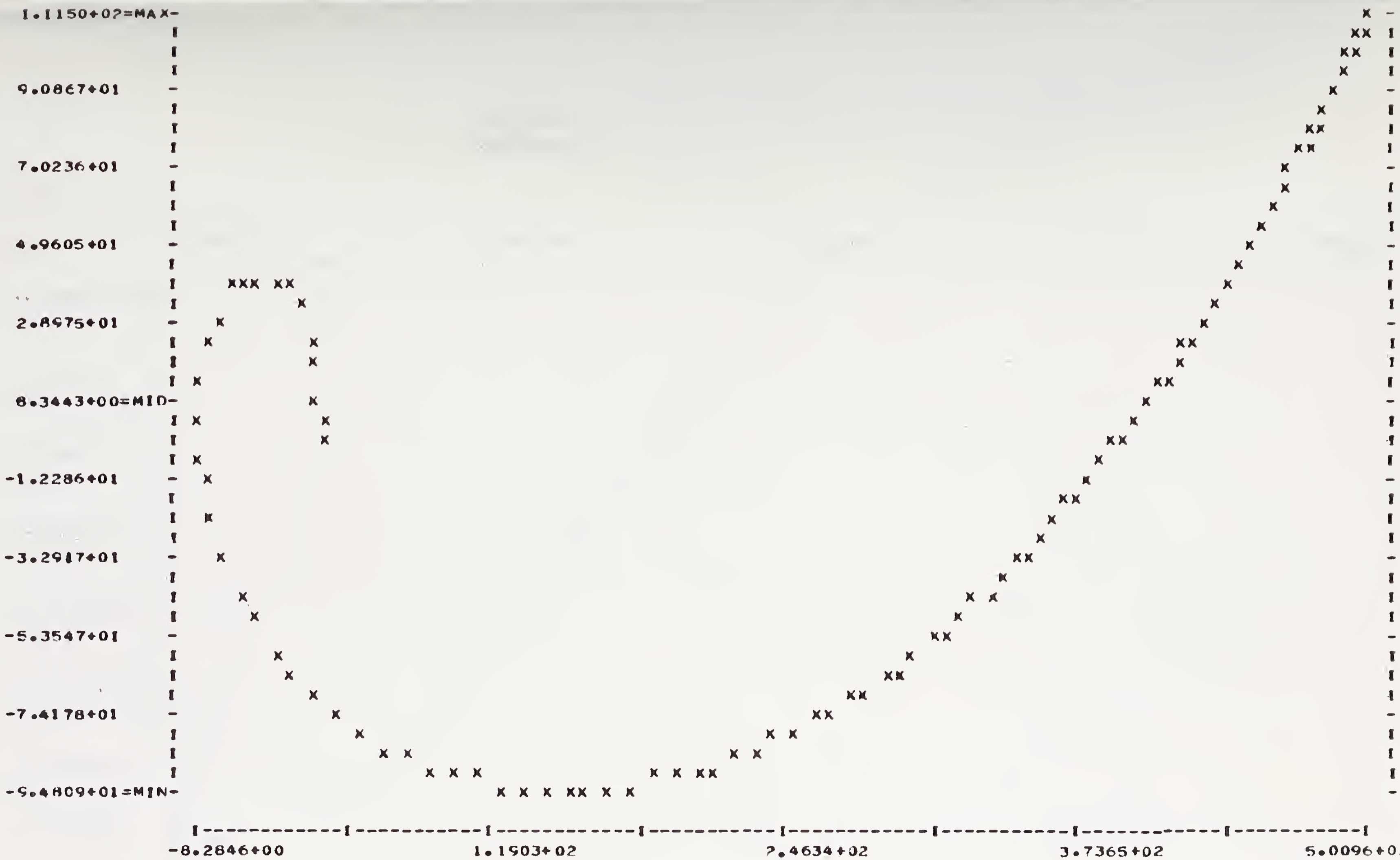
REASON. FINAL TIME HAS BEEN REACHED

RESULTS RELEVANT TO WHOLE RUN.

MAXVXP = 1.0121+02
MAXVYP = -9.8577+01
MAXVZP = 8.5224+01
MAXWVP = 1.1499+02



ORDINATE. Z
ABSCISSA. Y



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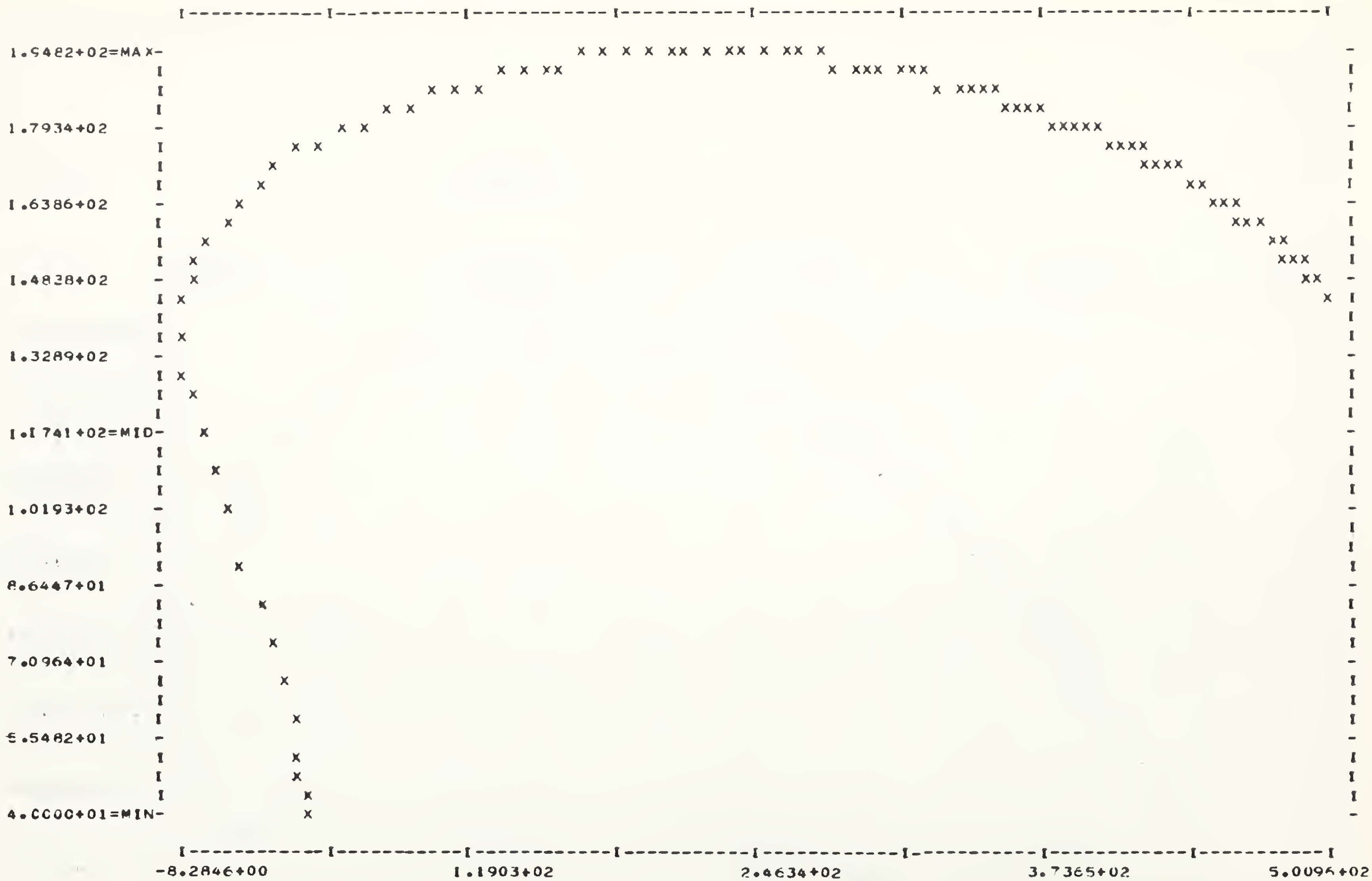

ORDINATE. Z
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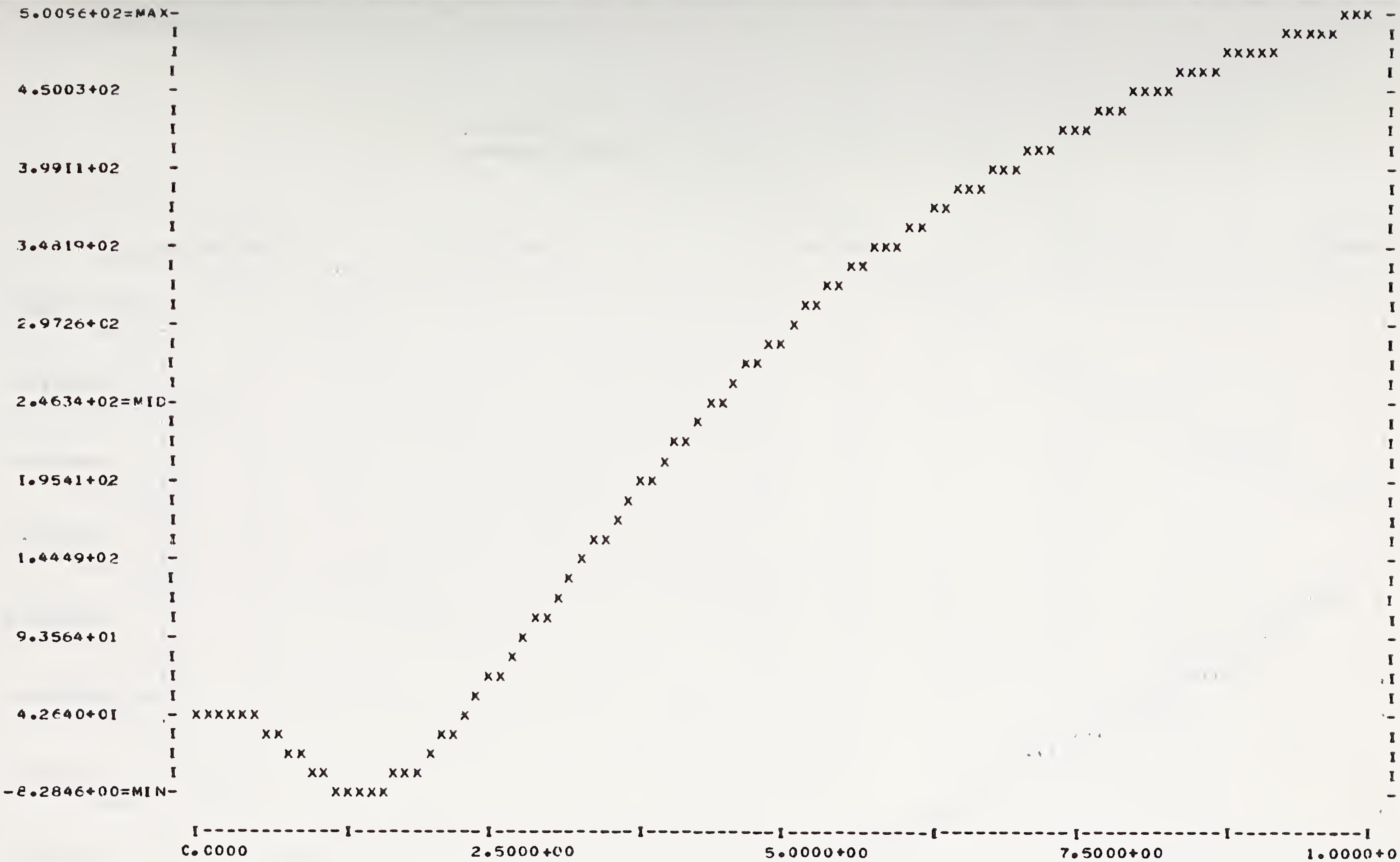
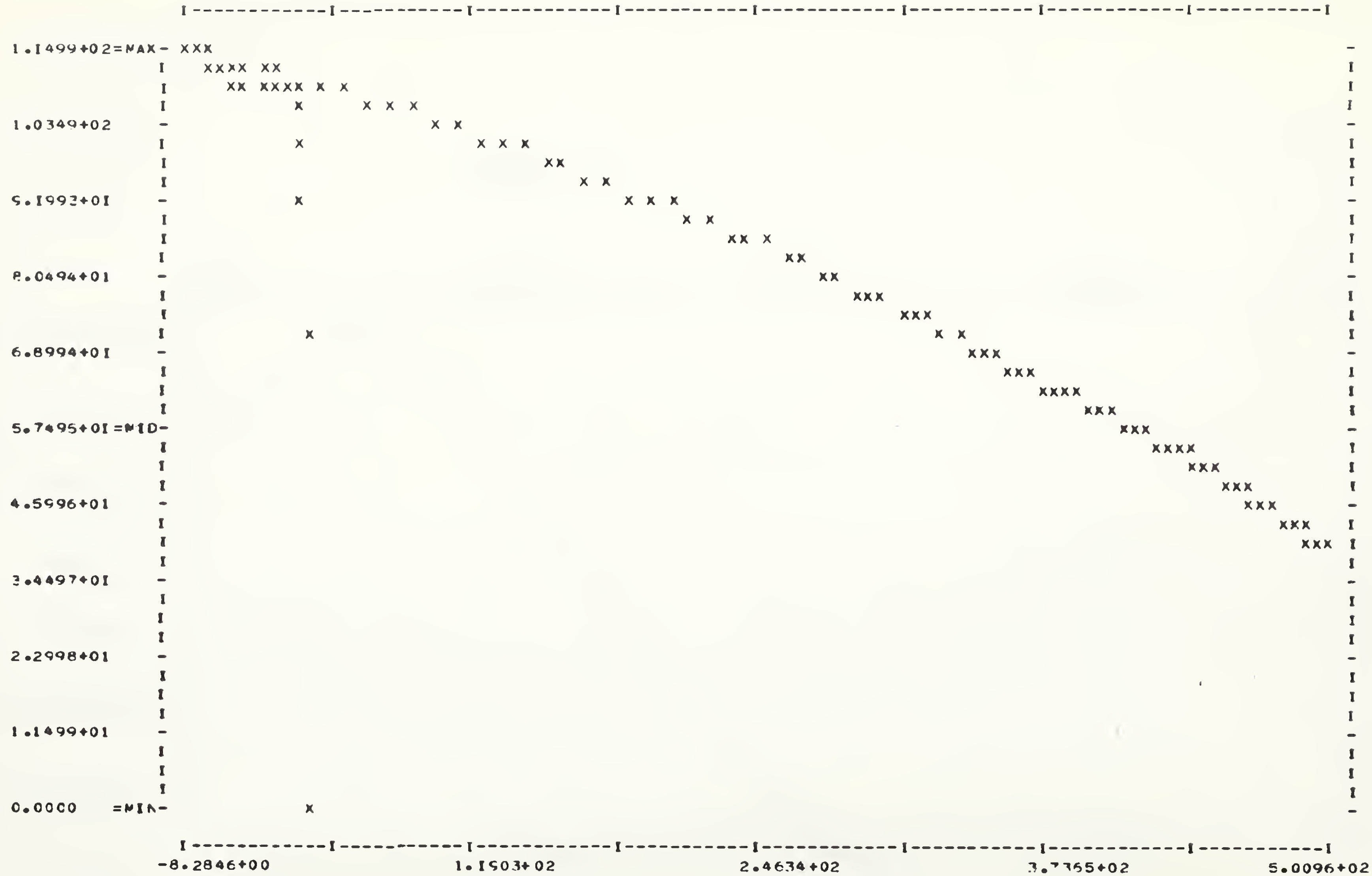
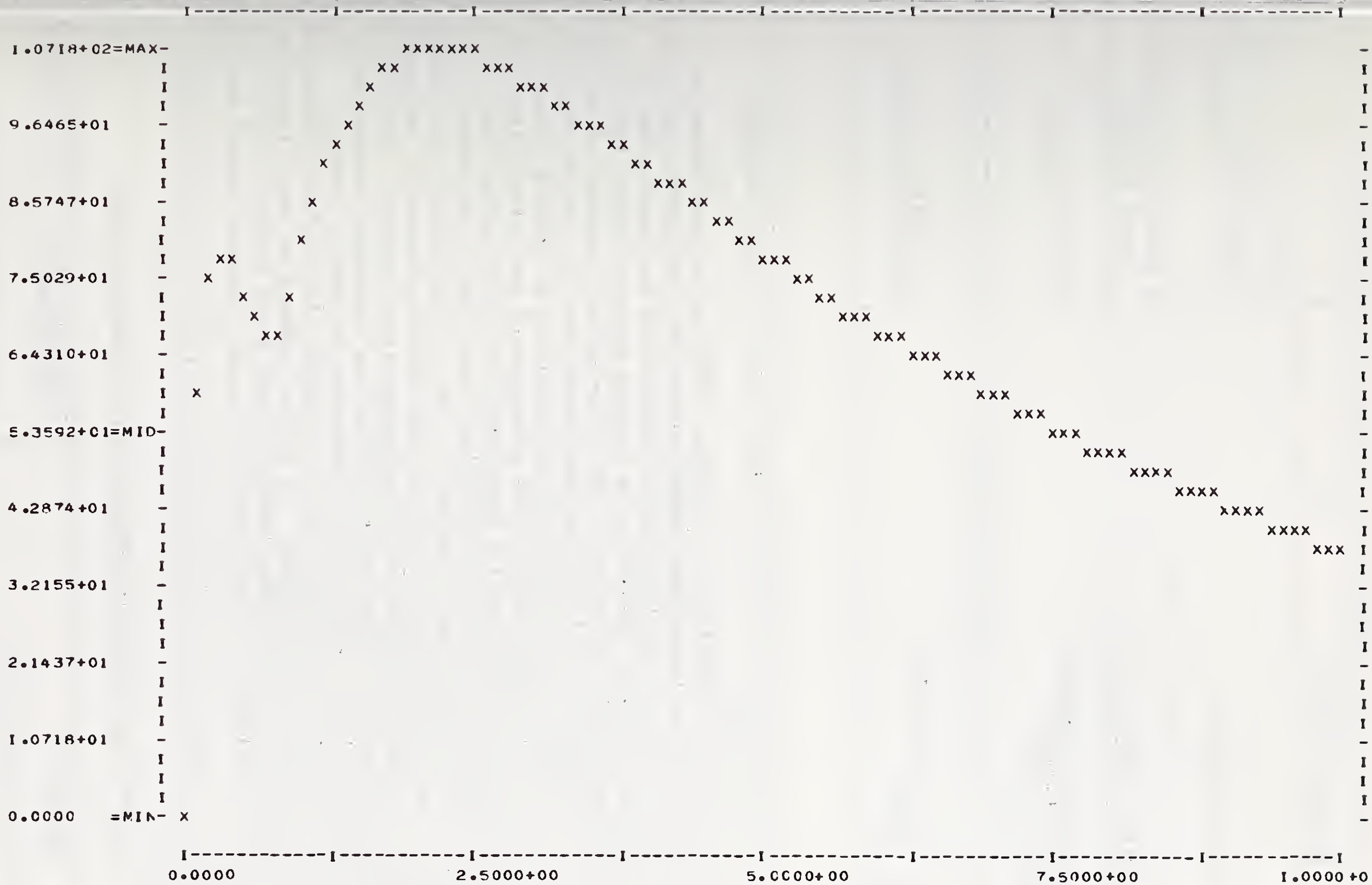


ORDINATE. X
ABSCISSA. TIME



ORDINATE. SPEED
ABSCISSA. X



ORDINATE. HORIZONTAL SPEED
ABSCISSA. T

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

At the request of the U.S. Nuclear Regulatory Commission (NRC)the National Bureau of Standards (NBS) has carried out an independent investigation into the question of tornado-borne missile speeds, with a view to assisting NRC in identifying pertinent areas of uncertainty and in estimating credible tornado-borne missile speeds - within the limitations inherent in the present state of the art. The investigation consists of two parts: 1) a study, covered in this report, in which a rational model for the missile motion is proposed, and numerical experiments are carried out corresponding to various assumptions on the initial conditions of the missile motion, the structure of the tornado flow, and the aerodynamic properties of the missile; 2) a theoretical and experimental study of tornado-borne missile aerodynamics, conducted by Colorado State Univ. (CSU) under contract with NBS, to be covered in a separate report by CSU. In the present report, the factors affecting missile motion, and their influence upon such motion, are examined. Information is provided on a computer program developed for calculating missile speeds. Maximum speeds for a number of specified potential tornado-borne missiles are presented, corresponding to a set of assumptions believed by the writers to be reasonable for design purposes. It is pointed out that higher speeds are conceivable if it is assumed that certain circumstances, examined in the body of the report, will obtain. It is the judgment of the writers that the probabilities of occurrence of such higher speeds for any given tornado strike are low. More than qualitative estimates of such probabilities are, however, beyond the scope of this investigation.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Missiles; nuclear engineering; structural engineering; tornadoes; wind.

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